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MEMORANDUM REPORT BRL-MR-3418

NUMERICAL RESOLUTION CALCULATION FOR
ELASTIC-PLASTIC IMPACT PROBLEMS

Bertina M. Creighton

December 1984

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elastic-plastic impact calculations in two dimensions. EPIC-2 makes use of triangular elements in several configurations. The results presented herein should serve as a guide to proper selection of mesh size and orientation for accurate representation of physical problems.

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I. INTRODUCTION

The finite element method has been used in the modeling of many problems concerning elastic-plastic impact behavior. The correct geometric representation of a mesh is important in obtaining accurate results without introducing numerical instabilities into the calculations. Here, triangular elements formulated for wave propagation and shock problems have been applied to both a classical elastic wave propagation problem and to the analysis of a system subjected to high velocity impact loading.

The objective of this study is to ascertain the effects of different orientations of the triangular elements and to determine the optimal mesh size and shape using EPIC-2, a computer code for elastic-plastic impact calculations in two dimensions, to best fit the solution. The information presented here is intended to be of assistance to potential users faced with applying the finite element method to impact problems and considering how to obtain meaningful solutions in two dimensions.

Wave propagation in the impact of a semi-infinite elastic bar was chosen because of the availability of a closed form solution. The analytical solution obtained by Richard Skalak,¹ and the elementary one-dimensional theory for elastic wave propagation in long, slender rods² were used to judge the convergence of the numerical solution in the various calculations. After obtaining the optimal mesh size, the effects upon the changes in artificial viscosity factors were observed.

A large deformation problem was also considered, in the form of a sphere impacting a flat plate at high velocity, to assess effects of grid size and orientation on deformations, pressures, and strains in the presence of large plastic flow.

II. APPROACH

Two numerical examples are provided to demonstrate the effects of mesh orientation and artificial viscosity upon the numerical solution.

All computational results were obtained with the EPIC-2 (Elastic-Plastic Impact Calculation in Two Dimensions) computer code,³ a Lagrangian finite

¹ Skalak, R., "Longitudinal Impact of a Semi-Infinite Circular Elastic Bar," *Trans. ASME, J. Applied Mech.*, Vol 24, Dec 1955, pp. 59-63.

² Johnson, W., *Impact Strength of Materials*, Crane, Russak and Co., New York, 1972, pp. 1-12.

³ Johnson, G. R., "EPIC-2, A Computer Program for Elastic-Plastic Impact Calculations in Two Dimensions Plus Spin," Honeywell, Inc., Defense Systems Division, Contract Report ARBRL-CR-00373, June 1978 (AD# A058786).

element formulation where constant strain triangles are used to represent a continuum, and the hydrostatic pressure in a given element is computed using the Mie-Gr nisen equation of state. The code treats problems involving wave propagation and elastic-plastic flow. It is arranged to provide solutions for projectile-target impacts and explosive detonation problems.

Material properties used for the numerical computations are given in Table 1.

Table 1. Material Properties

	Steel (Mar-M-300)	Aluminum (7075-T6)
Density (kg/M ³)	7855.00	2784.00
Shear Modulus (GPa)	79.57	27.51
Yield Stress (GPa)	2.07	0.29
Ultimate Stress (GPa)	2.07	0.32

The propagation of elastic waves in a steel bar has been studied to ascertain the effects of mesh orientation and artificial viscosity on the numerical solution by comparison with analytical solutions. The analytical solutions assume a semi-infinite bar. The calculations presented herein are for a long bar, length-to-diameter ratio of 50. Comparisons between the computational results and the analytical solutions are made before the arrival of any reflected waves from the rear surface. Thus, the finite length bar modeled was effectively a semi-infinite bar at the times at which the comparisons are made. The elastic impact simulation involves a bar of circular cross-section made of high-hard steel. The length of the bar is 12.7 cm and the diameter is 0.254 cm. The bar impacts a rigid wall with a striking velocity of 3.048 m/sec.

In the numerical simulation of the elastic bar, a grid of quadrilateral elements comprised of four triangles per quadrilateral was used. Various spatial discretizations, such as element aspect ratio and the number of elements across the bar's radius, have been studied. The elementary one-dimensional wave equation and the approximate theory for determining how elastic stress waves propagate in infinite bars, given by Richard Skalak, are compared with the computational results in Section III.

The elementary one-dimensional linear wave equation is

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial z^2}, \quad (1)$$

where u is the displacement along the z axis of the bar. The constant, c , is the elementary wave velocity for a bounded medium and is given by

$$c = \sqrt{\frac{E}{\rho}} \quad (2)$$

and the exact dilatational wave velocity in an infinite elastic medium is

$$c = \sqrt{\frac{E(1 - \nu)}{(1 - \nu)(1 - 2\nu)\rho}}, \quad (3)$$

where ν is Poisson's ratio, E is Young's Modulus, and ρ is the density. According to the elementary theory, a pulse propagates along the bar without changing its shape. The general form of the wave solution is

$$u = f(z - ct) + g(z + ct). \quad (4)$$

The specific shape of the wave is determined by the form of f . Since the wave velocity, c , is constant for an ideal one-dimensional elastic medium, the shape of the wave does not change during propagation. The equation of motion given in (1) is valid for wave propagation in slender bars (length-to-diameter ratio of at least 10) and neglects the effects of transverse strain, lateral inertia, body forces, and dissipative forces. However, the equation can be applied with reasonable success to problems where the longest wavelength is six to ten times greater than the typical cross-sectional dimension of the bar.

The analysis of the transmission of waves in a uniform circular cylinder of finite length is complex because of the introduction of the end conditions on the bar. Richard Skalak has obtained a solution, which includes transverse strain effects, for elastic wave propagation in a semi-infinite elastic bar subjected to a suddenly applied impulse through the superposition of two parts, namely

$$\frac{\partial^2 u_r}{\partial t^2} = \frac{(\lambda\mu + 2\mu)}{\rho} \frac{\partial \Delta}{\partial r} + 2\mu \frac{\partial \omega_0}{\partial z}, \quad (5)$$

$$\frac{\partial^2 u_z}{\partial t^2} = \frac{(\lambda + 2\mu)}{\rho} \frac{\partial \Delta}{\partial z} - \frac{2\mu}{r} \frac{\partial}{\partial r} (r\omega_0), \quad (6)$$

where

$$\Delta = -\frac{1}{r} \frac{\partial(rU_r)}{\partial r} + \frac{\partial U_z}{\partial z}, \quad (7)$$

$$\omega_0 = \frac{1}{2} \left(\frac{\partial u_r}{\partial z} - \frac{\partial u_z}{\partial r} \right), \quad (8)$$

and

$$c^2 = \frac{\lambda + 2\mu}{\rho}. \quad (9)$$

Skalak's solution is valid for late times at points far away from the point of application of the load and is obtained in terms of Airy's integral. The approximate solution is of the form

$$u_z' = \frac{-v}{c} \left[\frac{1}{6} + \int_0^{\alpha'} \text{Ai}(\alpha) d\alpha + \frac{1}{6} + \int_0^{\alpha''} \text{Ai}(\alpha) d\alpha \right], \quad (10)$$

where v is Poisson ratio, α' and α'' are the non-dimensional coordinates which measure the distance at any point along the wavelength, with respect to velocity, and $\int \text{Ai}(\alpha) d\alpha$ is Airy's integral with respect to α , a function of time. Figure 1 depicts Skalak's refinements to the elementary theory solution. These solutions provide a guide to the reliability and accuracy of the numerical simulations.

When dealing with numerical solutions to exact problems, difficulties arise when shock waves are present. Von Neuman and Richtmyer first introduced artificial viscosity⁴ in order to treat problems involving physical and mathematical discontinuities in a continuum. Their one term formulation,

$$q_1 = c_0^2 \rho (\Delta x)^2 \left(\frac{\partial \dot{x}}{\partial x} \right)^2 \quad (11)$$

where $q_1 = 0$ for $\partial \dot{x} / \partial x$ greater than or equal to zero smears the shock front over several mesh widths, thus changing the discontinuities into steep but continuous wavefronts which can be treated in the context of continuum mechanics.

Later on, it was found that many finite difference discretizations tended to introduce spurious wave motions into the numerical computations. In order

⁴ Wilkins, M. L., "Use of Artificial Viscosity in Multi-Dimensional Fluid Dynamic Calculations," *Journal of Comp. Physics*, Vol 36, No. 3, July 15, 1980.

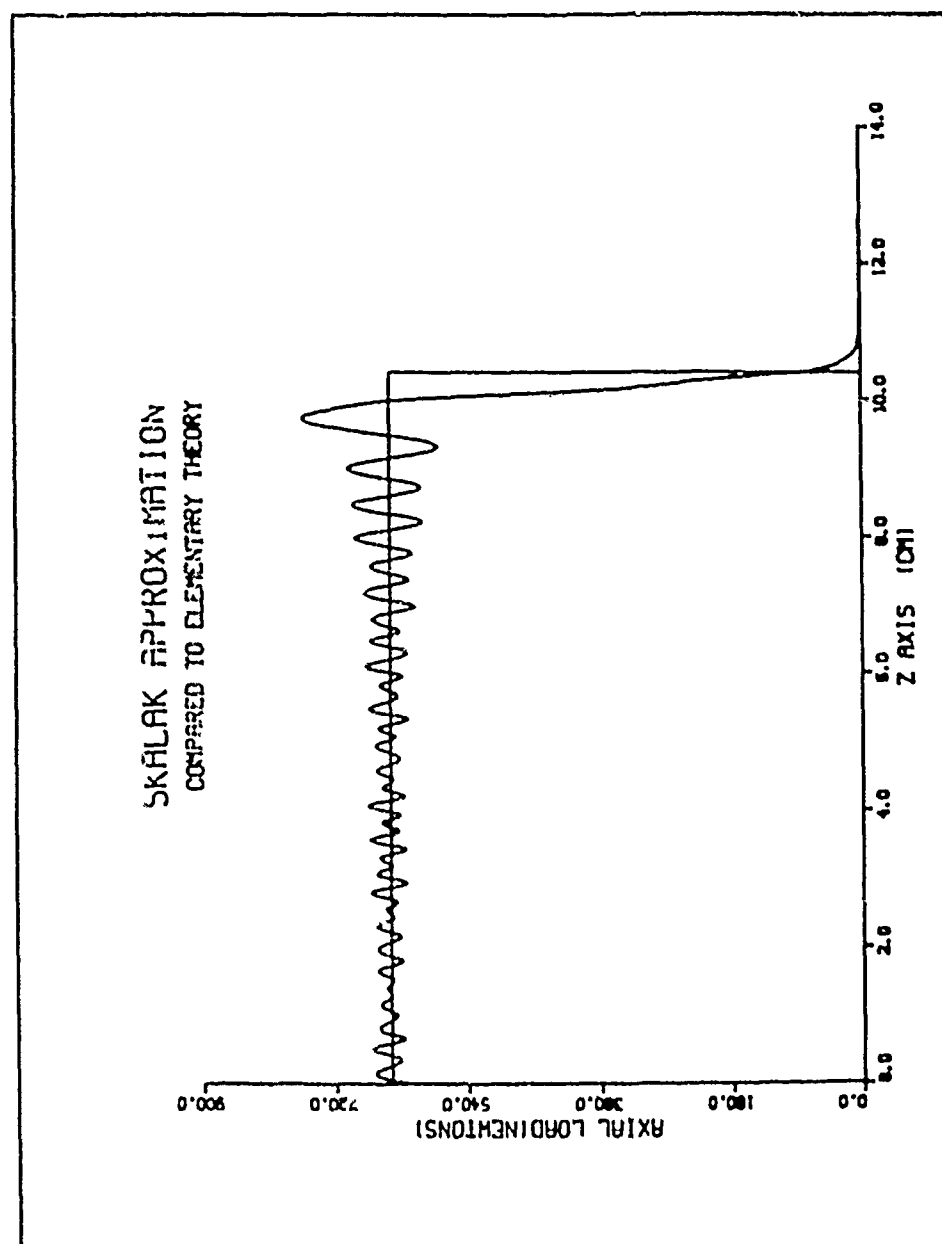


Figure 1. Richard Skalak's Analytical Solution.

to suppress these, the artificial viscosity concept of Von Neuman and Richtmyer was extended to include a second term,

$$q_2 = \alpha_1 \frac{\partial \dot{x}}{\partial x} \quad (12)$$

$$\alpha_1 = (k_2)^2 \rho (\Delta x)^2 \left| \frac{\partial \dot{x}}{\partial x} \right| \quad (13)$$

and $q_2 = 0$ for $\partial \dot{x} / \partial x$ greater than or equal to zero. The main purpose of the second term was to suppress numerical noise as the shock wave propagates. For computations involving solids where the sound speed exists even at zero pressure, the linear and quadratic terms are used together. Hence, the generalized form is

$$q = k_1 \rho c \Delta x \left| \frac{\partial x}{\partial x} \right| - k_2^2 \rho (\Delta x)^2 \left| \frac{\partial x}{\partial x} \right| \left| \frac{\partial x}{\partial x} \right|, \quad (14)$$

where c is the sound speed, Δx is the characteristic dimension in the direction of wave propagation, ρ is the density, k_1 is the linear component, k_2 is the quadratic component, and x is the coordinate in the direction of motion. In EPIC-2, the generalized formula is added to the equation of state, which helps to eliminate numerical overshooting and lower level noise arising from discontinuities.

Having established the effects of irregular meshing and artificial viscosity on problems of elastic wave propagation, an additional set of simulations was undertaken to obtain an appreciation of the effect of these parameters on situations involving plastic flow. Only the mesh orientation was varied in the plastic impact study. Three mesh orientations were considered which are shown in Figure 2.

The effects of the various mesh orientations can be seen from the deformation patterns but become even more evident when strain and pressure distributions are studied. Finally, while not explicitly considered in this study, the behavior of numerical results in the presence of different failure criteria and post-failure models can be inferred from the results presented.

III. RESULTS

A. Elastic Impact

A comparison of the results from the various computations to determine the grid size, using quadrilateral elements comprised of four triangles, gave

A. IDIA = 1 Hypotenuse of the Triangle is drawn from the lower left hand corner to the upper right hand corner.

B. IDIA = 2 Hypotenuse of the Triangle is drawn from the upper left hand corner to the lower right hand corner.

C. Quadrilateral Elements Comprised of four Triangles.

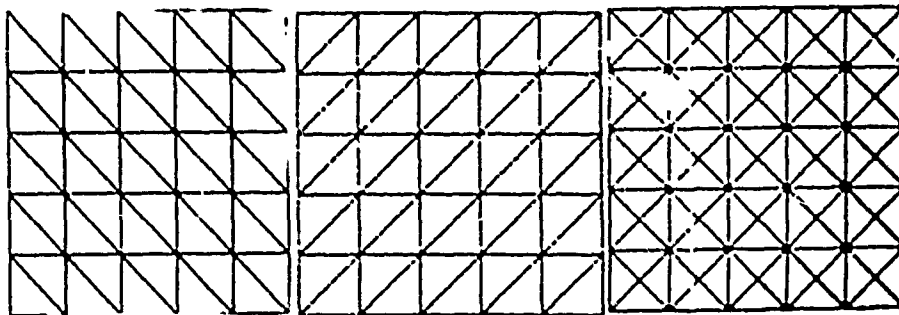


Figure 2. Element Orientations in SPID-2 Code.

some interesting results, as shown in Figures 3 - 14. In all six cases, the change in amplitude and time remained relatively small. Using a uniform grid and refining the mesh, there is one considerable effect: by increasing the number of nodal points and decreasing the size of the elements, the higher frequencies are resolved.

In Figures 4, 6 and 8 the wave profile in the bar at $20\mu s$ is presented for 1, 2 and 3 elements across the radius respectively. In all three cases the element aspect ratio was a constant of 1:1. Thus, as the number of elements across the radius is increased, the resolution is improved since the higher frequency waves are now being accounted for in the numerical solution. Comparison of the high resolution wave profile in Figure 8 with Skalak's solution in Figure 1 shows close agreement.

In contrast to the elementary theory solution, the wavefront predicted from both Skalak's solution and the numerical solution has a finite slope. The elementary theory reasonably approximates some features, such as peak amplitude of the behavior of long bars subjected to impulsive loading; but it is incapable of resolving some aspects of the problem. The solution implies that the waves transmitted are traveling with constant velocity and without alteration of shape. In the Skalak and numerical solutions, the oscillations arise due to transverse strain effects. They are not artificially introduced by the numerical scheme. Furthermore, as one refines the mesh, the wave profile exhibits an increase in the number of oscillations behind the wave front.

Figures 8, 10, 12 and 14 show the wave profile in the bar at $20\mu s$ for element aspect ratios of 1:1, 2:1, 3:1 and 4:1 respectively. As the aspect ratio is increased the axial resolution decreases. This decrease in axial resolution prevents the numerical solution from capturing the high frequency wave components in the axial direction. This can be observed in the plots by the decrease in the number of oscillations in the wave profile behind the wave front.

The cost of computations may often be a major factor in determining the grid size. With a large aspect ratio and a coarse grid, the CPU time and costs are relatively low. While the cost and the CPU time practically double with grid refinements. Thus, the user must make a trade off between accuracy and cost.

The effects of changes in artificial viscosity coefficients are shown in Figures 15 - 17. An increase in the linear component results in excessive distortion of the wave front and a decrease in the oscillations occurring along the wavelength. The effect of changes in the quadratic coefficient is small. This is primarily because the quadratic component dissipates more rapidly away from the shock than the linear component.

B. Plastic Impact

An additional set of simulations has been performed to gain an appreciation for the effect of element orientation on the numerical solution of impacts involving plastic flow. In this set of computations the mesh orientation, in the target only, was altered. All computations were

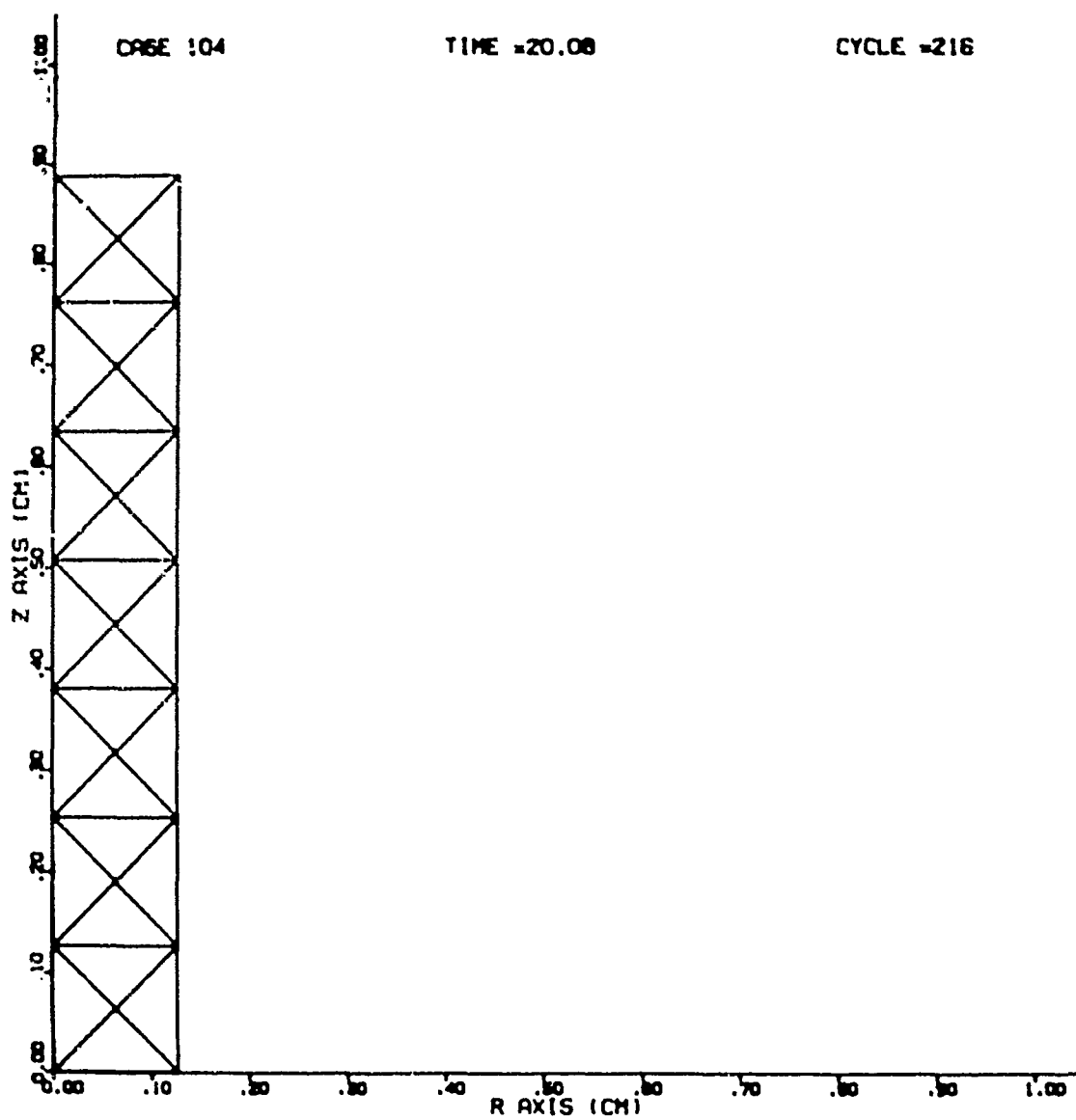


Figure 3. Computational Grid Map, 400 Elements.

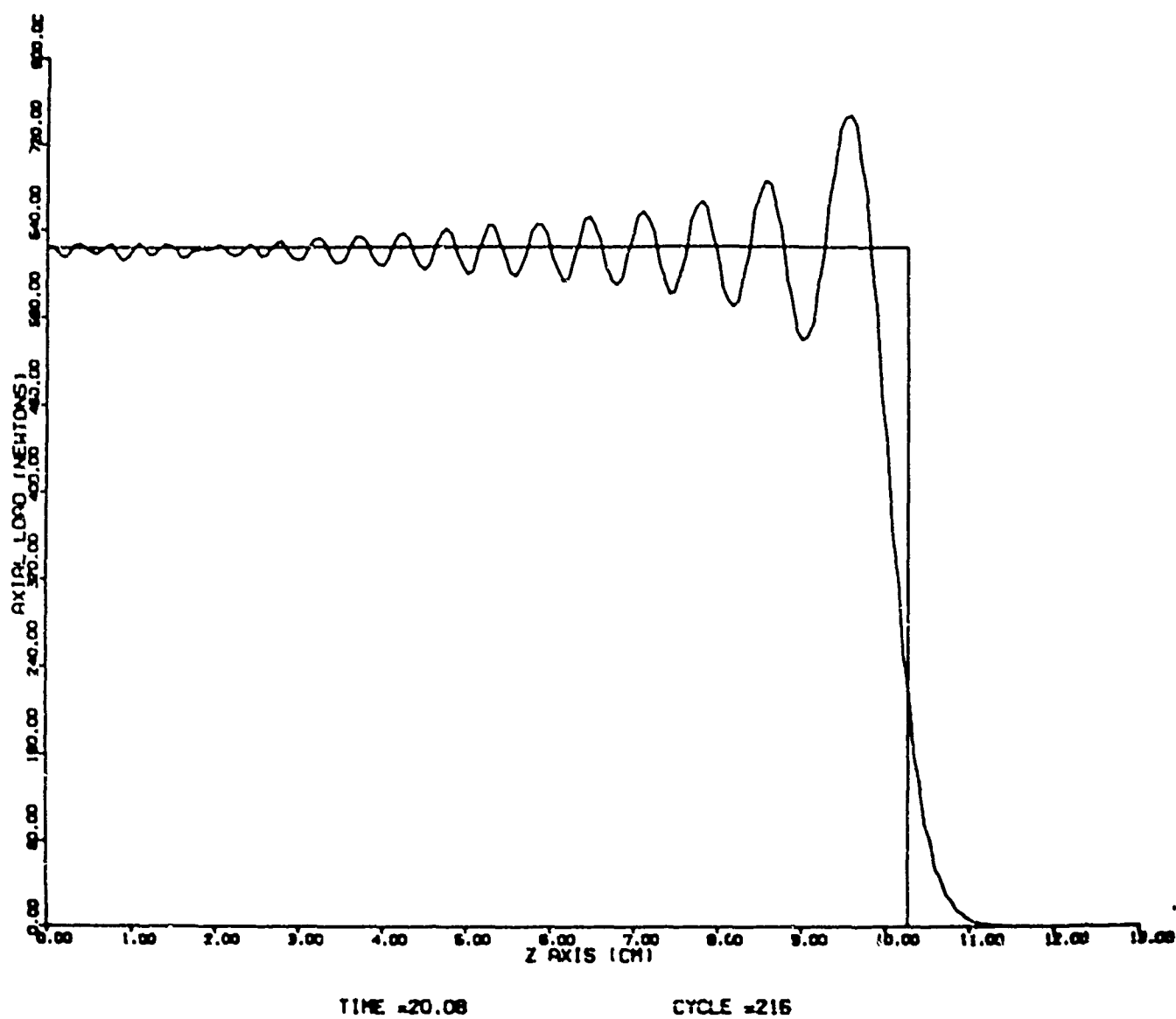


Figure 4. Load/Displacement in Bar, Aspect Ratio 1-1, Case 104.

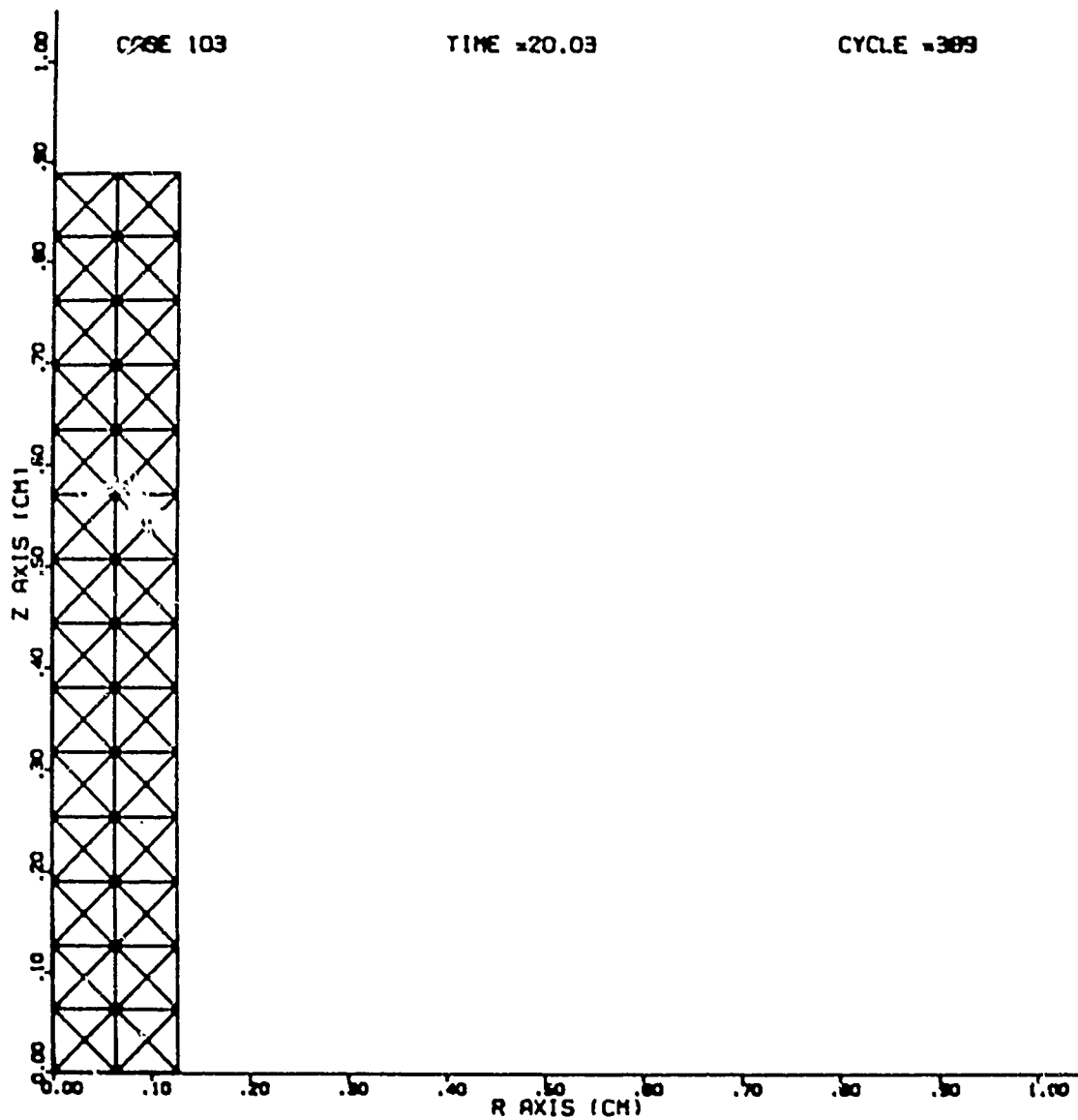


Figure 5. Computational Grid Map, 1600 Elements.

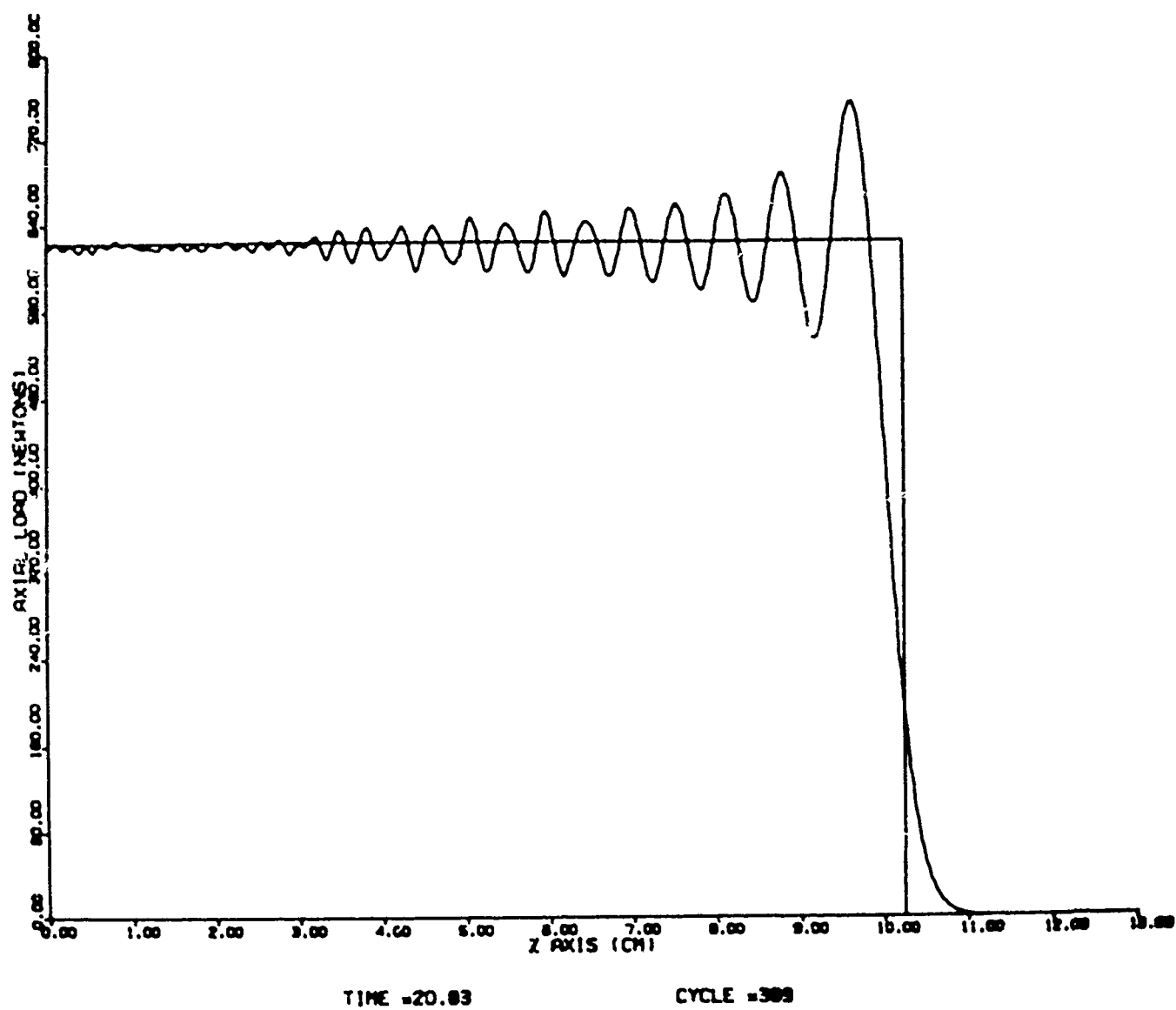


Figure 6. Load/Displacement in Bar, Aspect Ratio 1-1, Case 103.

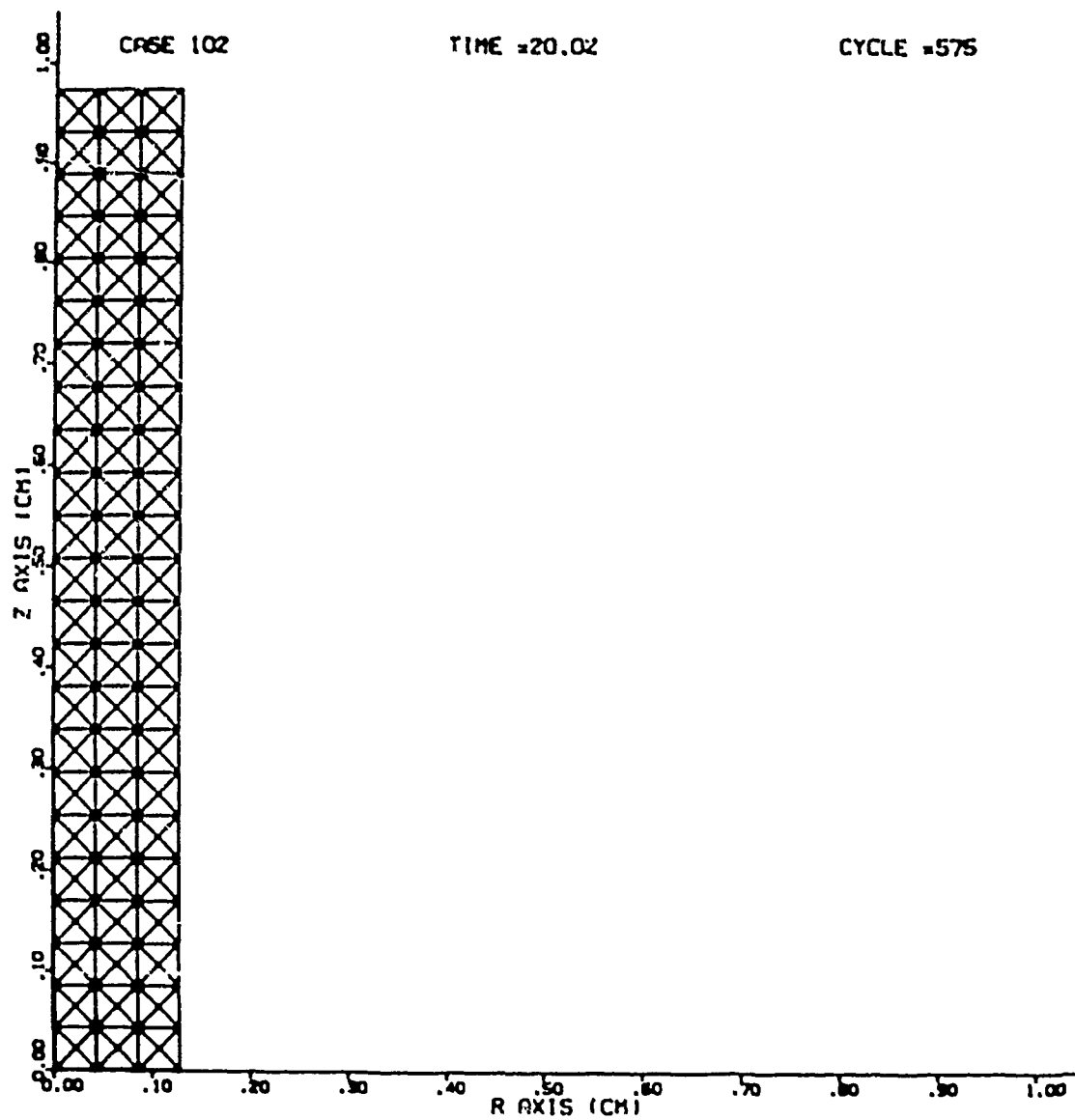


Figure 7. Computational Grid Map, 3600 Elements.

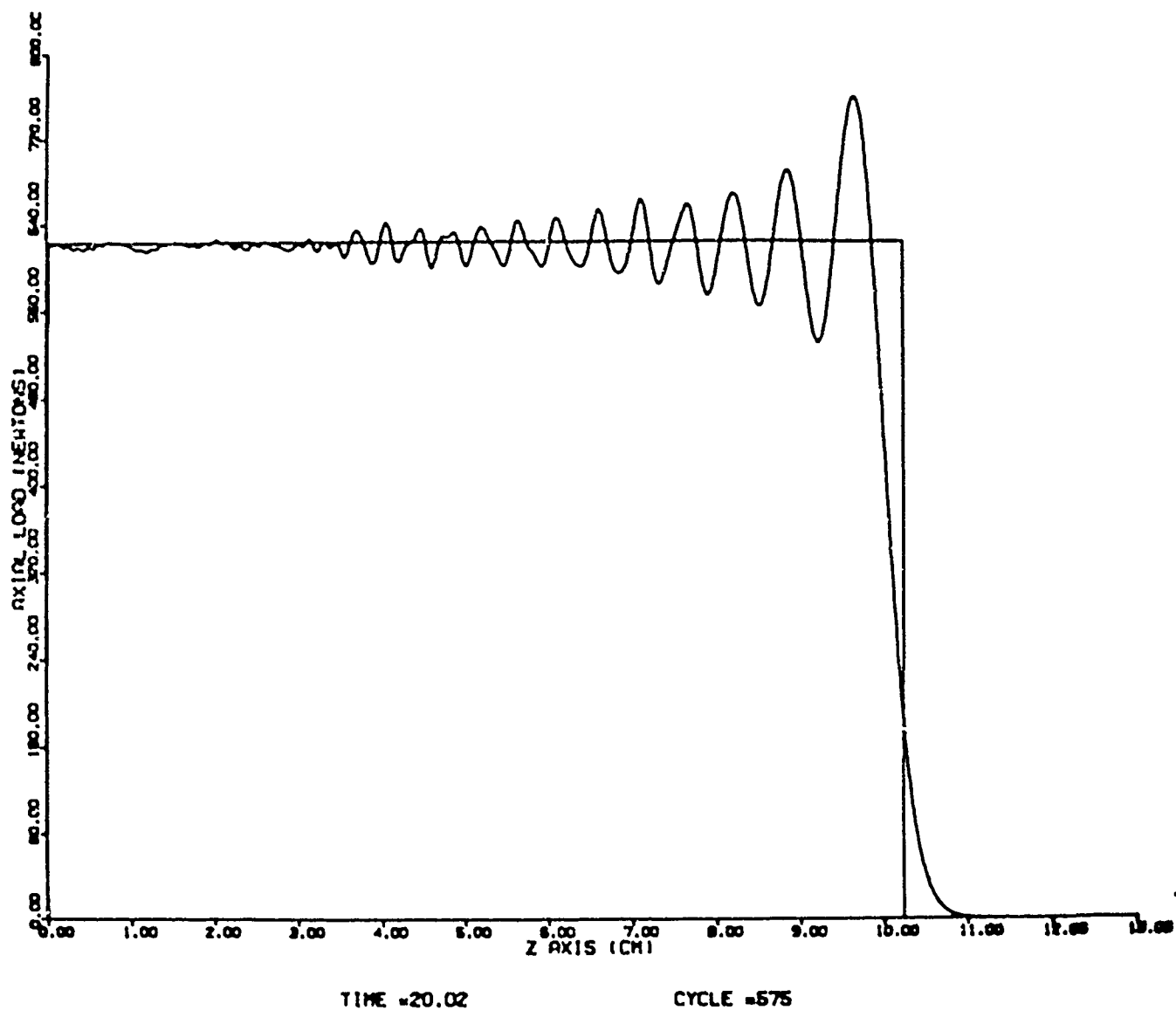


Figure 8. Load/Displacement in Bar, Aspect Ratio 1-1, Case 102.

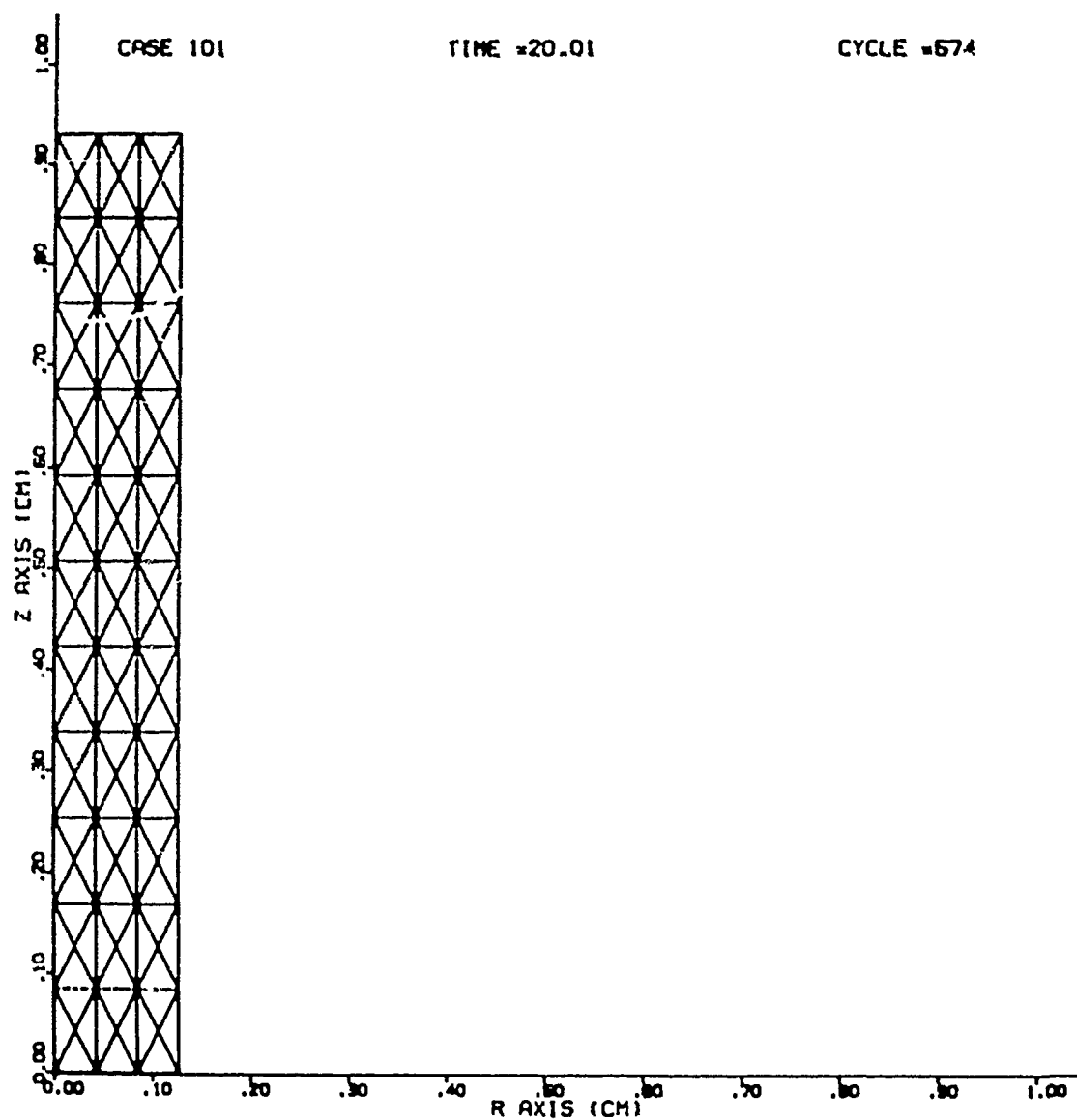


Figure 9. Computational Grid Map, 1800 Elements.

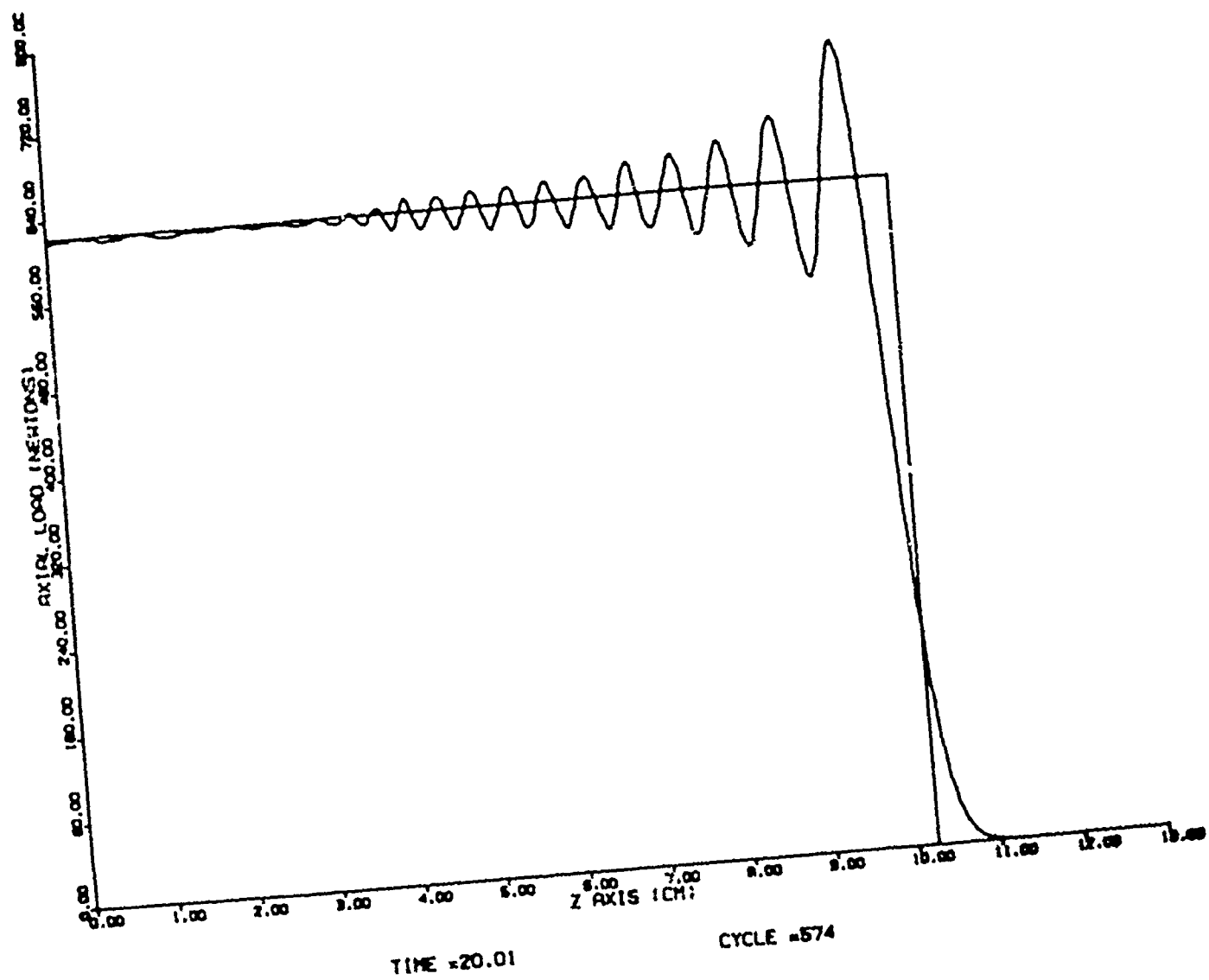


Figure 10. Load/Displacement in Bar, Aspect Ratio 2-1, Case 101.

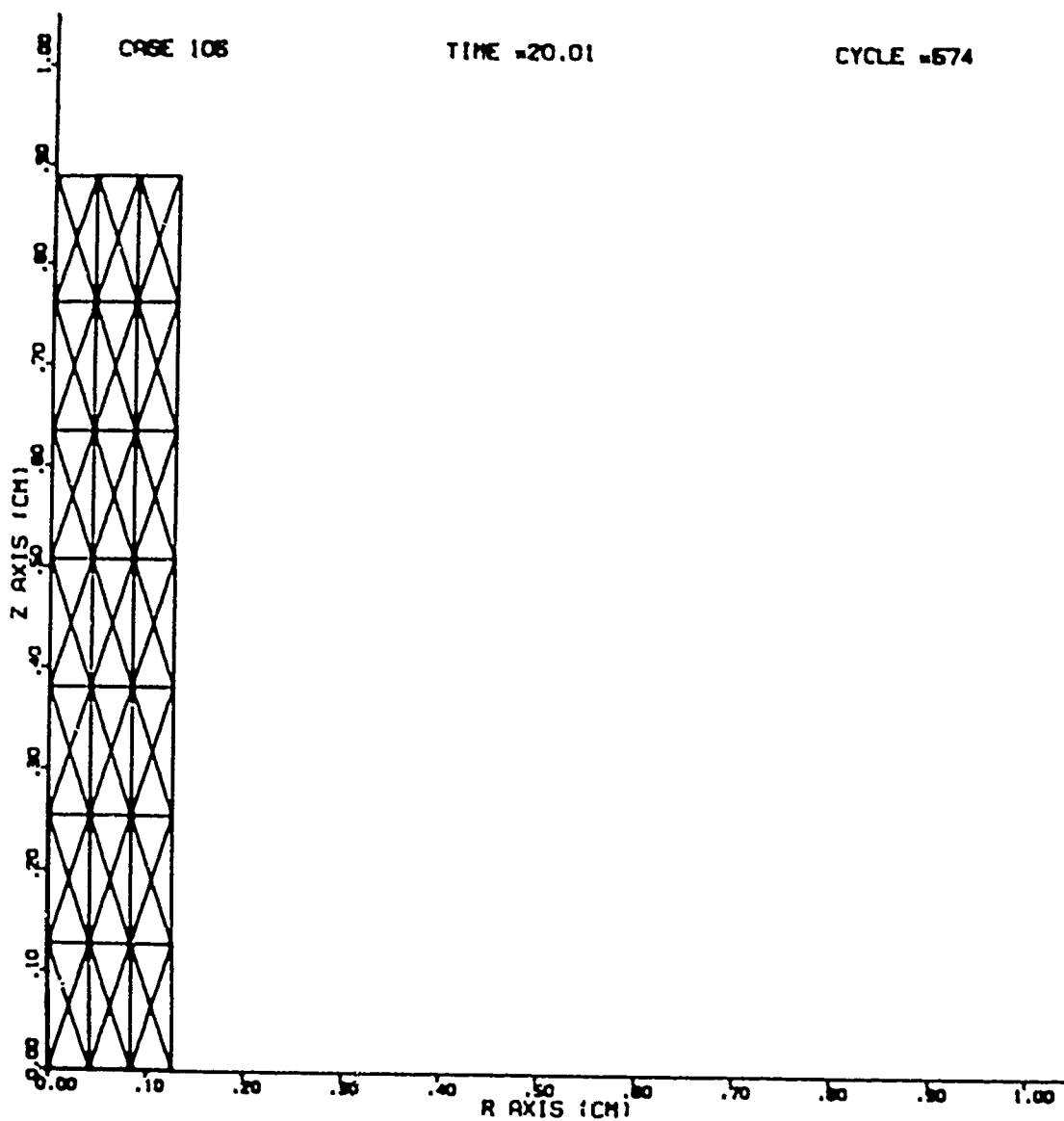


Figure 11. Computational Grid Map, 1200 Elements.

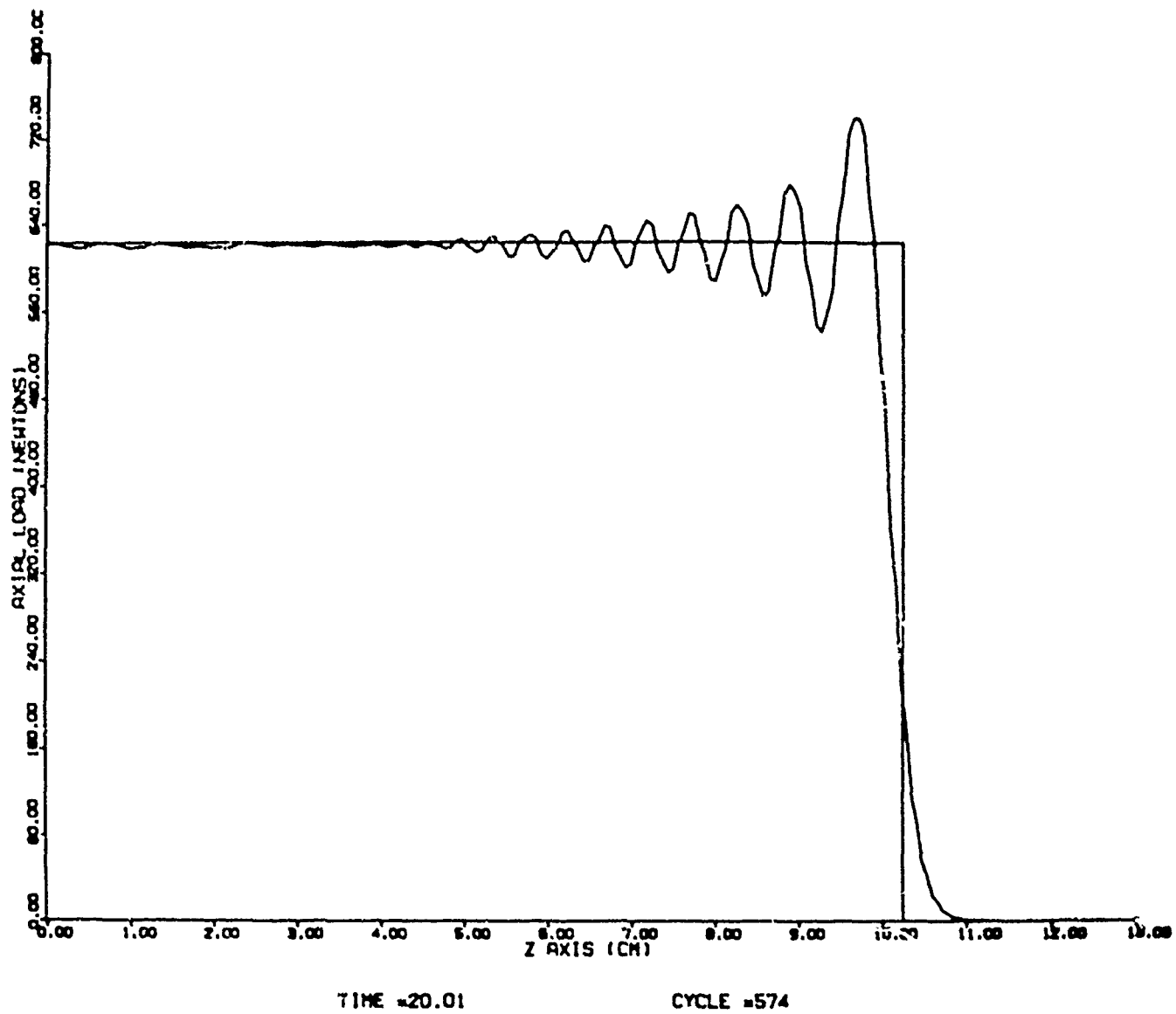


Figure 12. Load/Displacement in Bar, Aspect Ratio 3-1; Case 105.

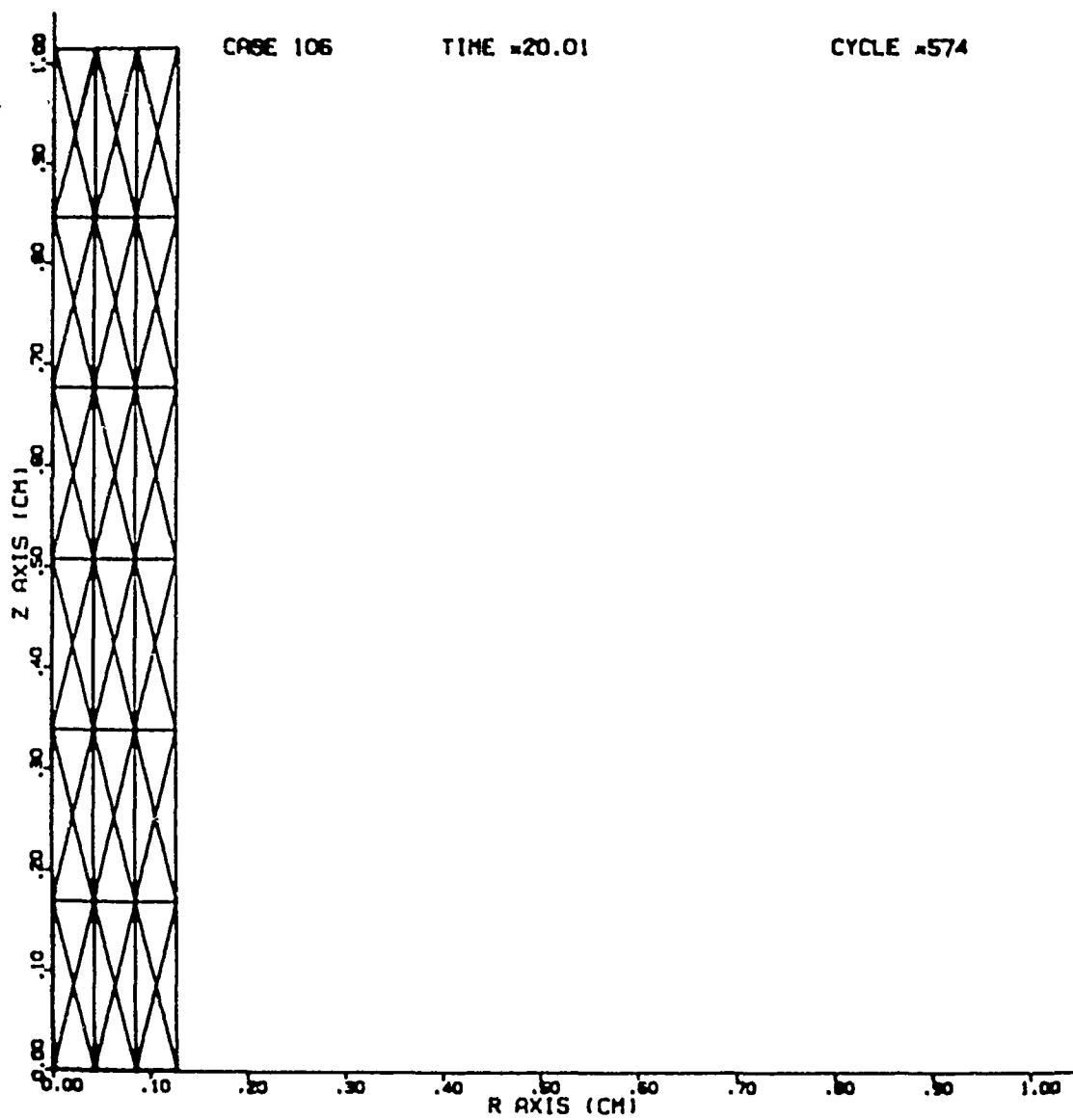


Figure 13. Computational Grid Map, 900 Elements.

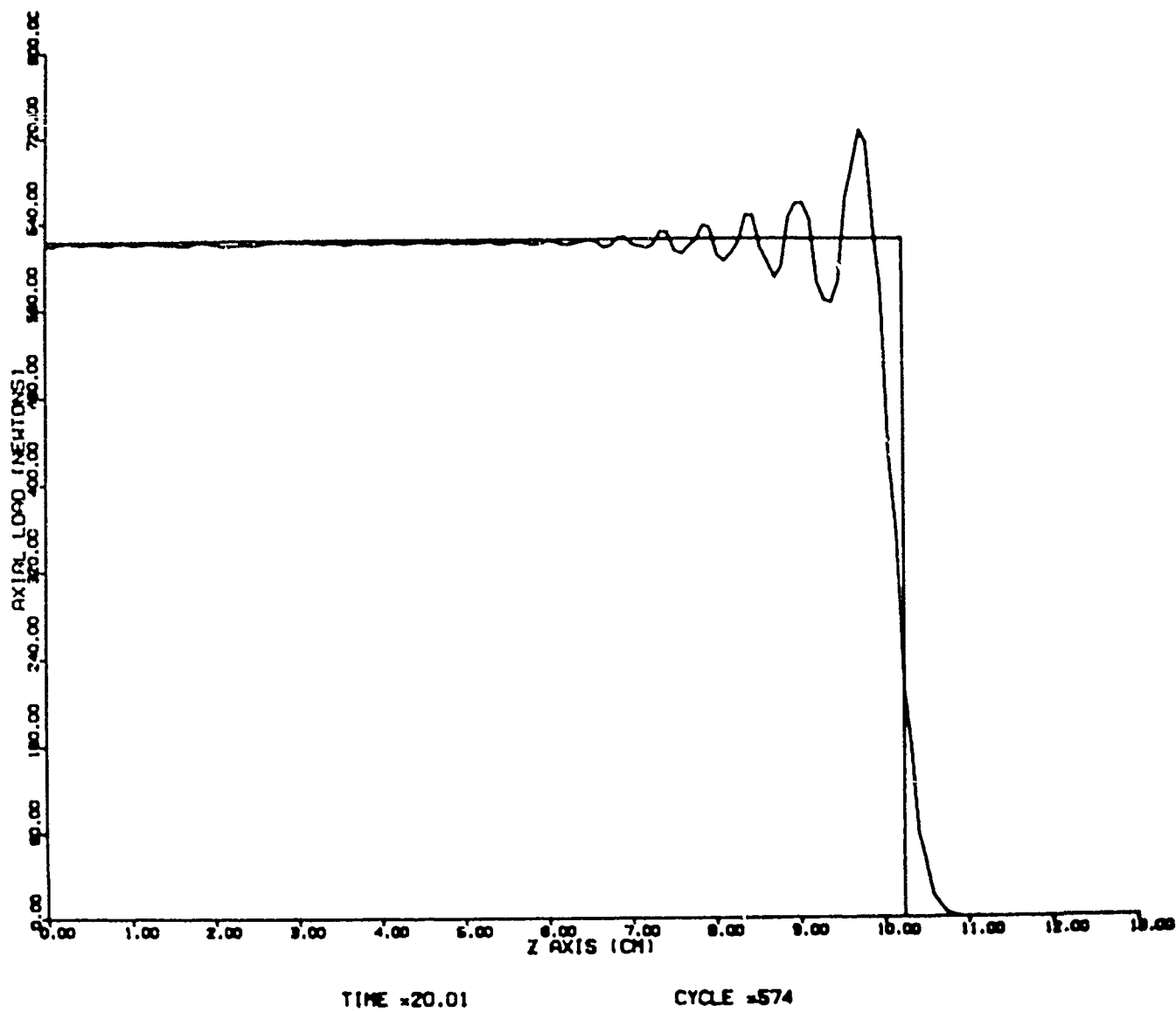


Figure 14. Load/Displacement in Bar, Aspect Ratio 4-1, Case 106.

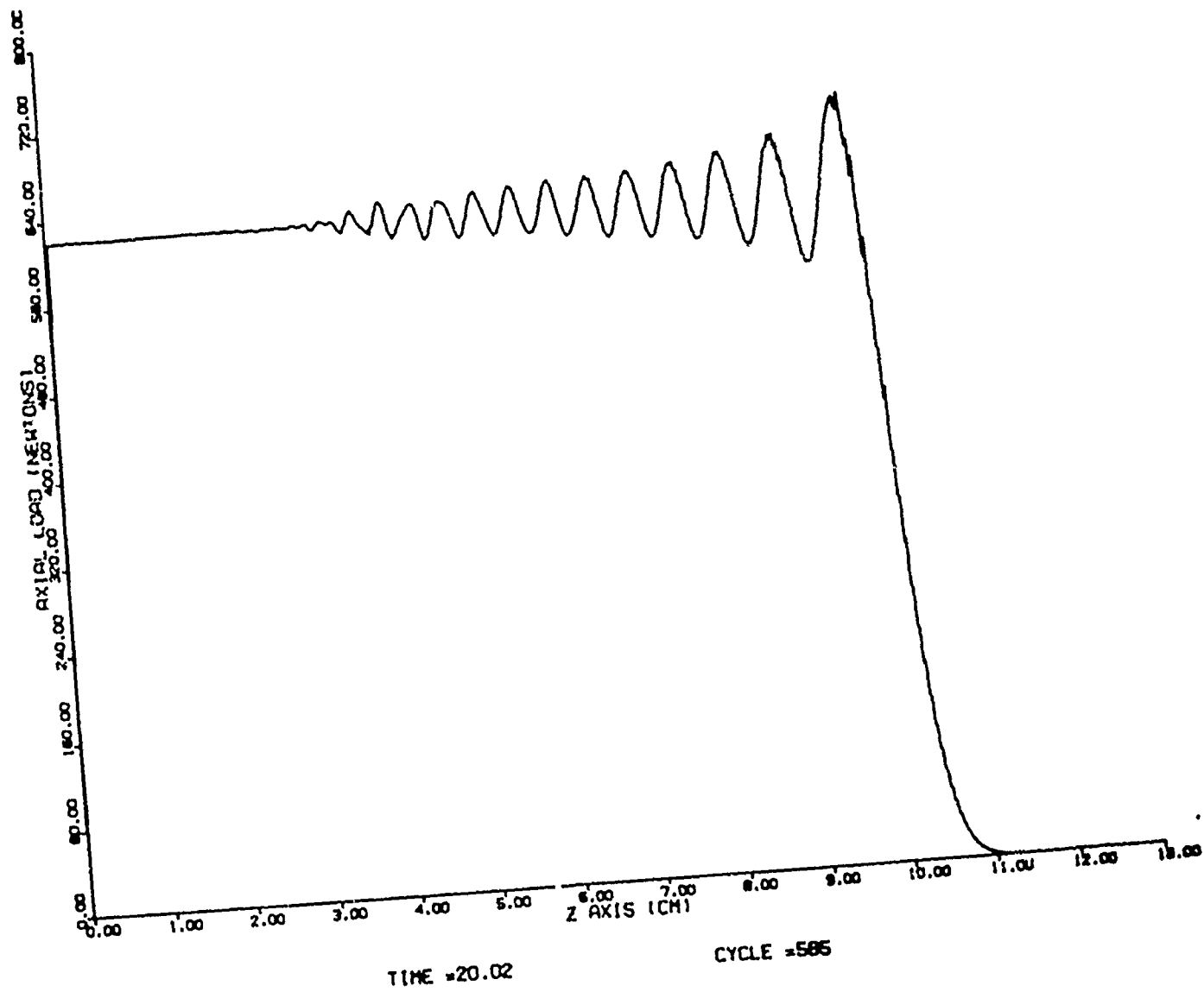


Figure 15. Artificial Viscosity Plot, Linear Factor 0.6, Quadratic Factor 4.0.

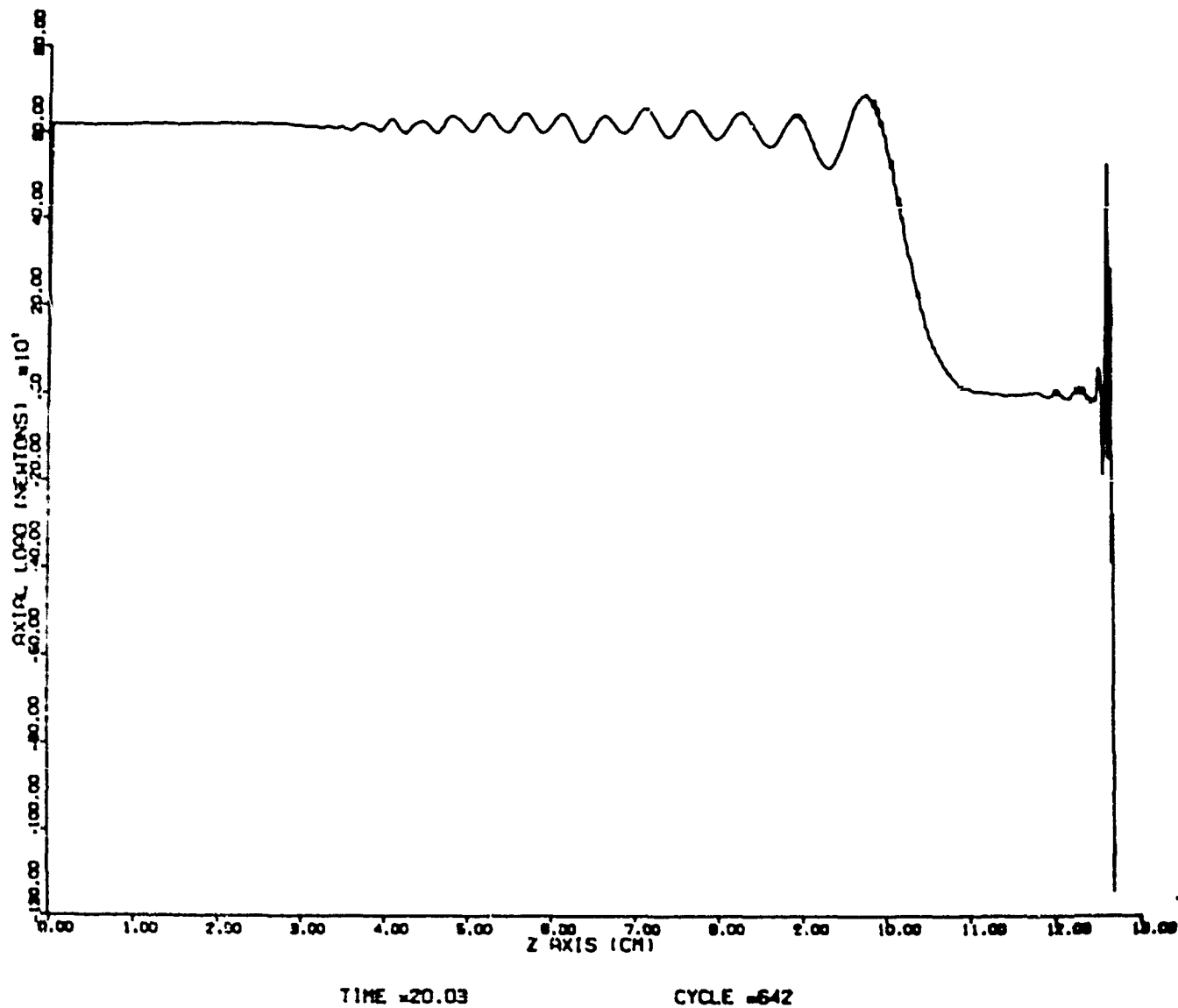


Figure 16. Artificial Viscosity Plot, Linear Factor 1.0, Quadratic Factor 4.0.

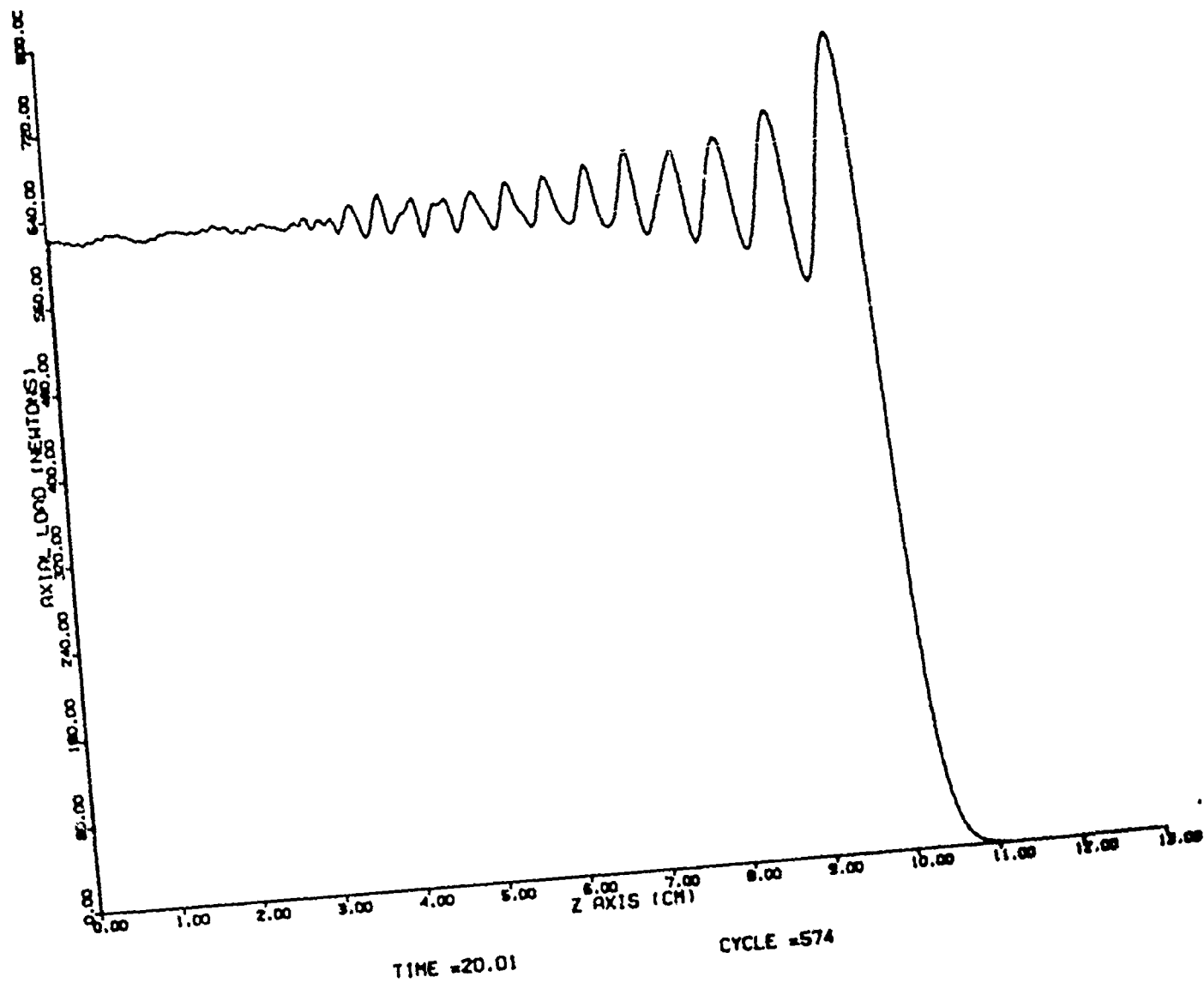


Figure 17. Artificial Viscosity Plot, Linear Factor 0.2, Quadratic Factor 2.0.

terminated at $10\mu\text{s}$, at which time the problems are sufficiently advanced to obtain reasonable results. The plastic impact studied consists of a steel sphere impacting an aluminum plate with a velocity of 1524 m/sec . The target plate thickness is 5.08 cm . The first target mesh orientation studied has the diagonals oriented with a positive slope ($\text{IDIA}=1$ in Figure 2), see Figure 18.

Figures 19 and 20 show the computational results at $5\mu\text{s}$ and $10\mu\text{s}$ respectively. At $10\mu\text{s}$ the target is forming a V-shape which will produce unrealistic deformations of the sphere and numerical instabilities. The computational results for the diagonals oriented with a negative slope are presented in Figures 21 and 22. At $5\mu\text{s}$ there is a slight pulling of the elements in a positive direction. By $10\mu\text{s}$ the target deformation is unreasonably stiff. Clearly, from Figures 19-22, the diagonal orientation has a significant influence on the predicted deformations.

In Figures 23 and 24 the quadrilateral elements comprised of four triangles allow for large deformations to occur without introducing artificial stiffness and/or asymmetries into the calculations, which are very typical of the triangular zones.

The response of the target material has been carefully examined for strain, pressure, and distribution as well as failure patterns. Material failure in the EPIC-2 code is governed by a failure criterion based on equivalent plastic strain. When a user's specified value for this quantity in an element is exceeded, the element is considered to have partially failed, and no tensile or shear stresses are permitted. In effect, the element behaves as a fluid. Total failure at higher values of equivalent plastic strain is also permitted. In this case, neither stress nor pressure can be carried by an element, and it is therefore ignored in the computational loop. The element connectivity is destroyed, but the nodes and their associated mass and velocity are retained to conserve mass and momentum. The results pertaining to the failure of the elements in the target are shown in Appendix A. Those elements marked with an "F" have exceeded the partial failure criterion of 0.6.

With regard to the first two cases, 107 ($\text{IDIA}=1$) and 108 ($\text{IDIA}=2$), the elements are being failed far away from the impact area. The total amount of failure is small; and the dispersement of the failed elements is more widely distributed in the target, in comparison to Case 109.

Element strains are derived by integrating the strain rates, which are obtained from the geometry of the elements and the nodal velocities. The strain contours are evaluated at five different levels and are shown in Appendix B. The strain contours in Cases 107 and 108 sharply deviate from the area of impact and extend out farther into the target in comparison to Case 109.

The hydrostatic pressure is dependent on the volumetric strains and internal energy in the elements. The results from the positive and negative pressure contours are shown in Appendix C. In comparing the first two Cases to the third, the positive pressure contours are more condensed around the area of impact and the failed elements. With regard to the negative pressure fields, there is a greater magnitude of the pressure in the projectile than in

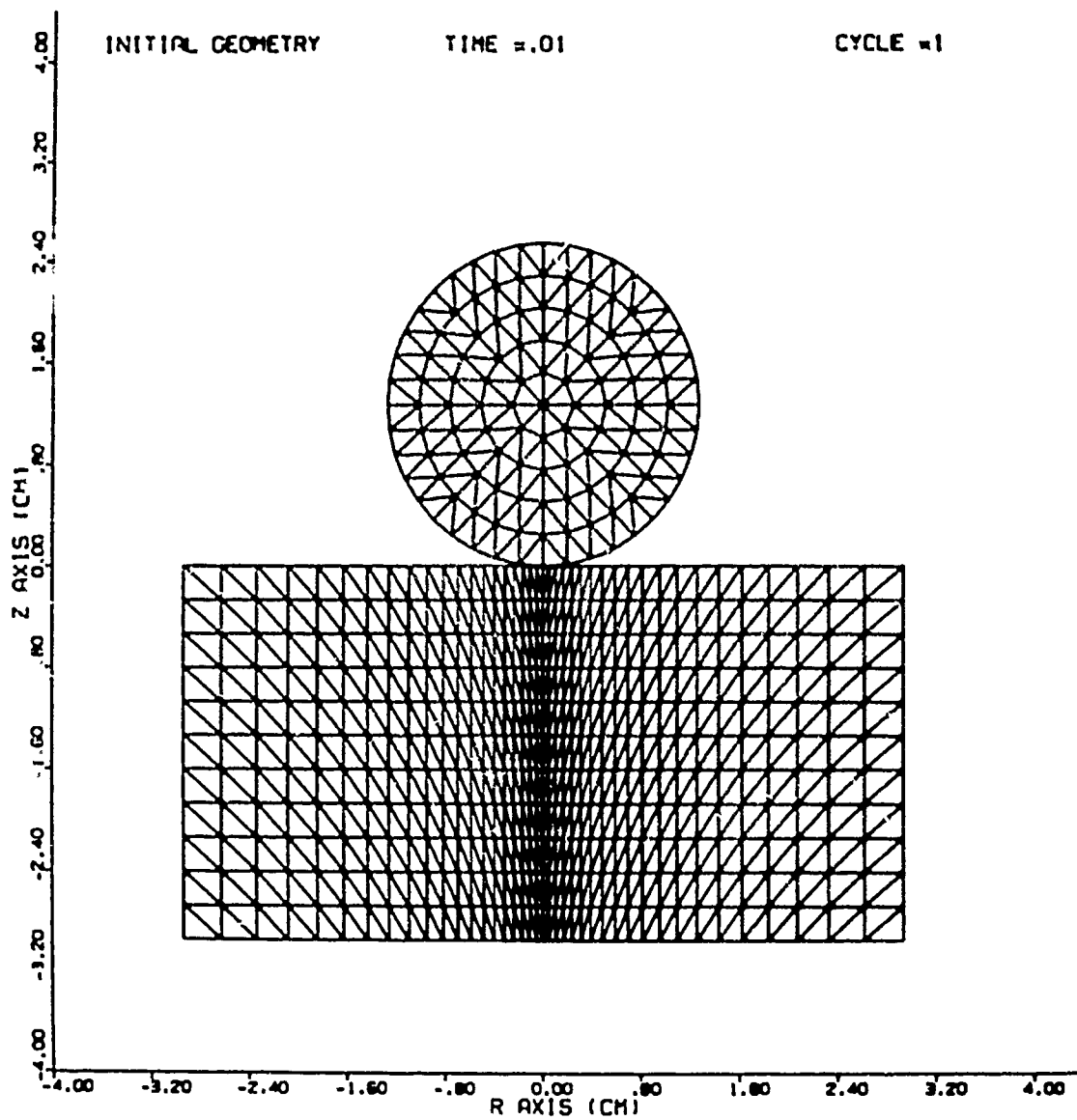


Figure 18. Initial Configuration of Sphere/Plate Impact.

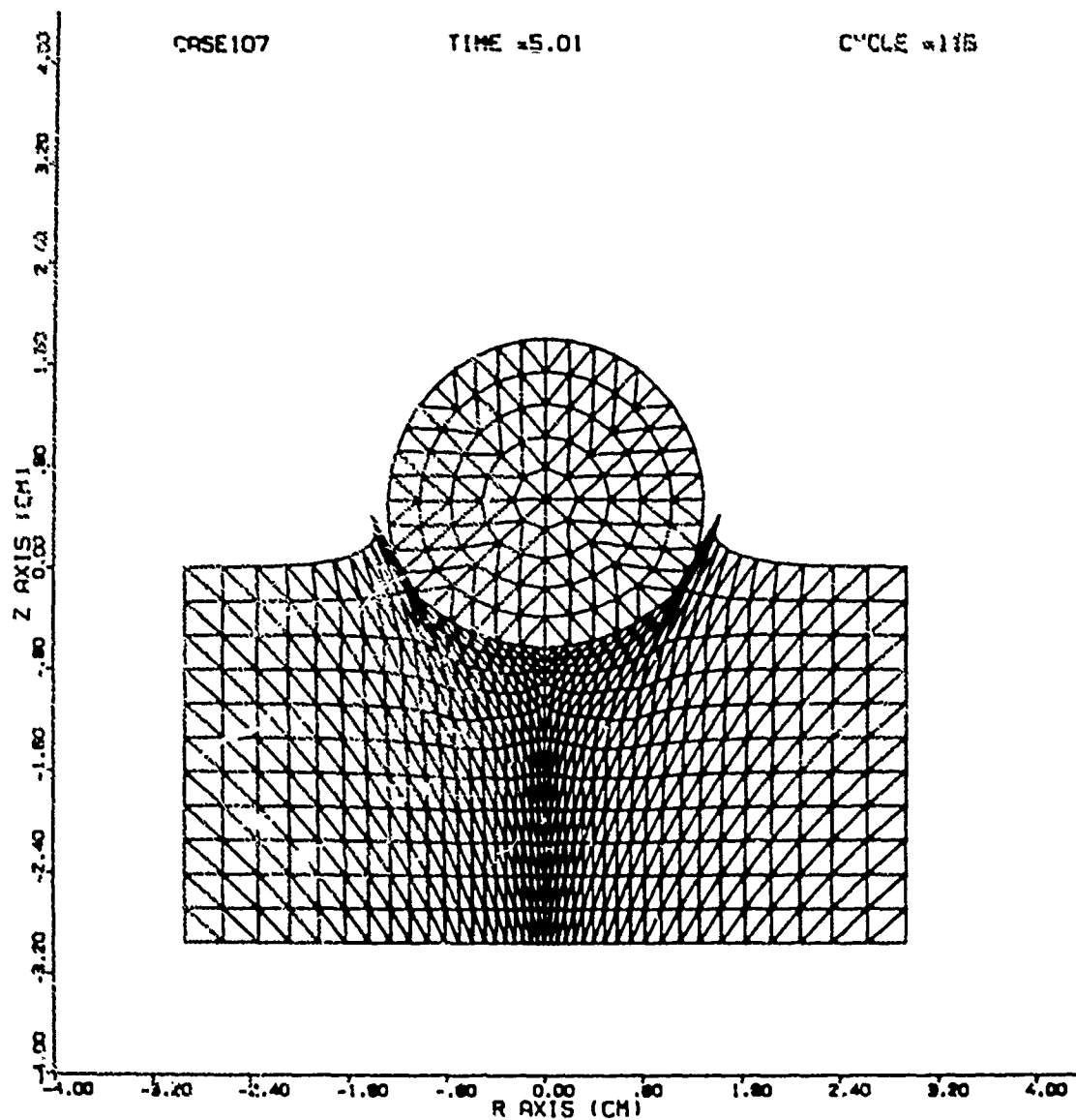


Figure 19. Deformation of Sphere/Plate Impact, $t = 5 \mu s$, IDIA = 1.

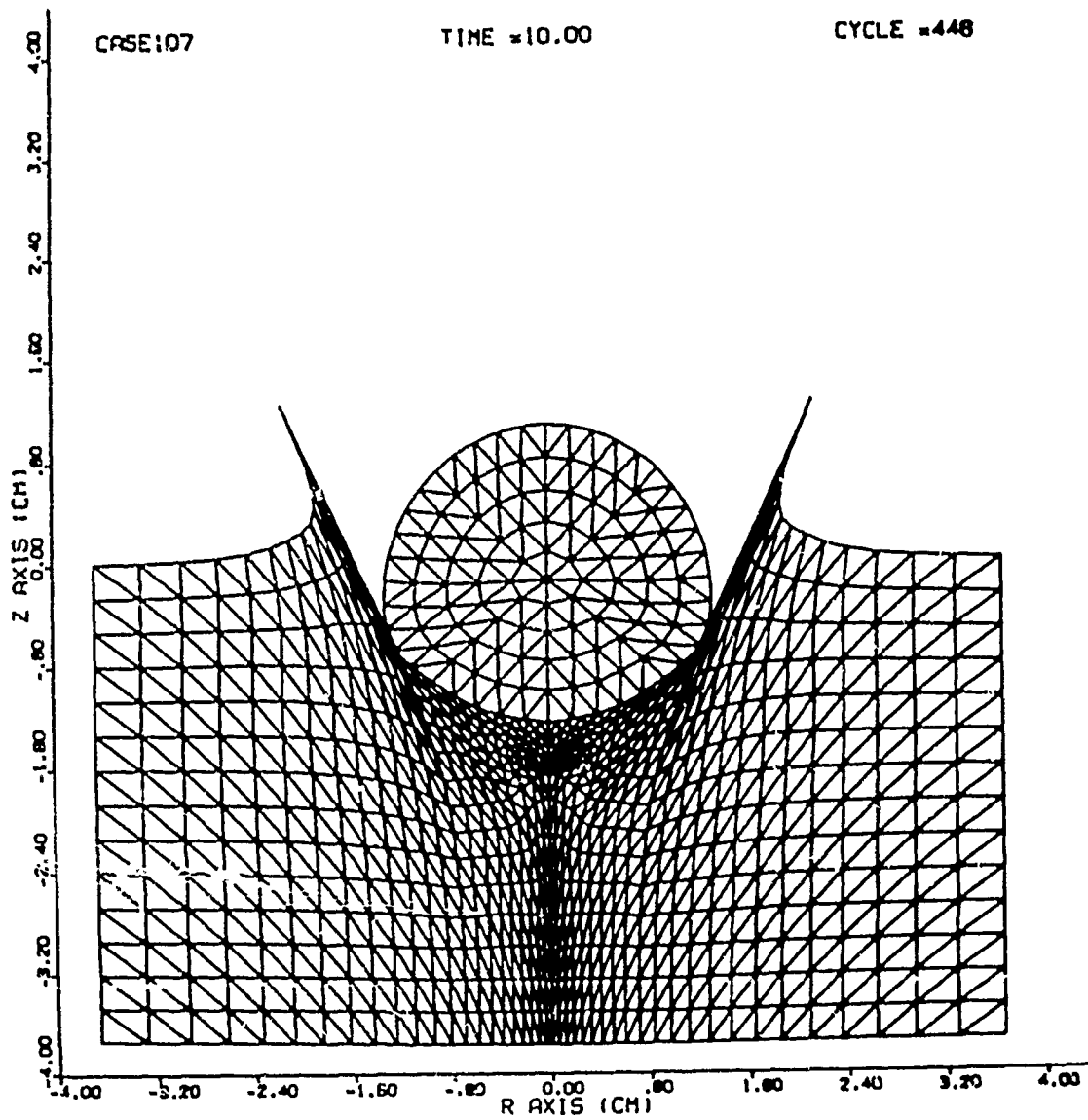


Figure 20. Deformation of Sphere/Plate Impact, $t = 10 \mu s$, IDIA = 1.

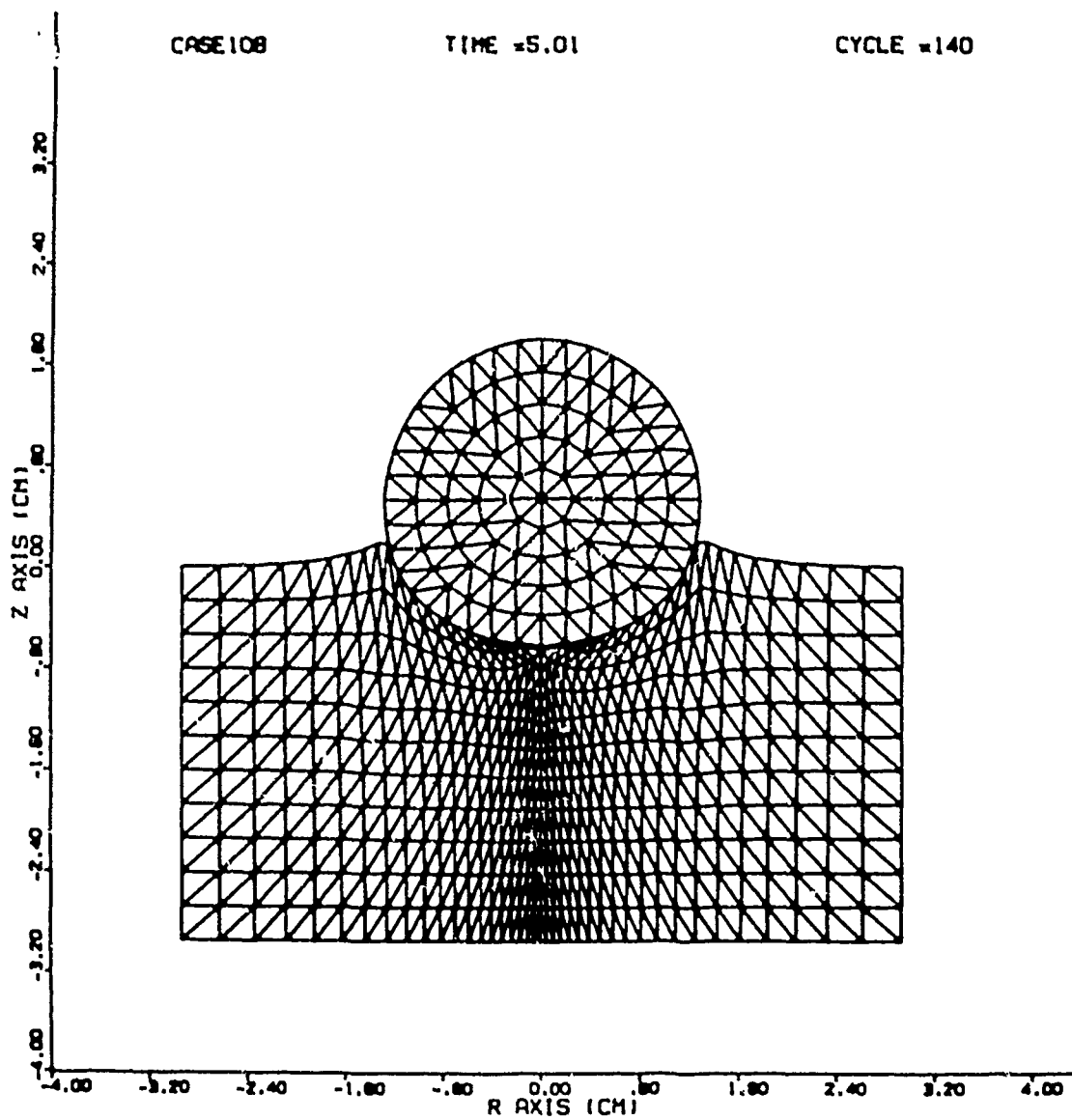


Figure 21. Deformation of Sphere/Plate Impact, $t = 5 \mu s$, IDIA = 2.

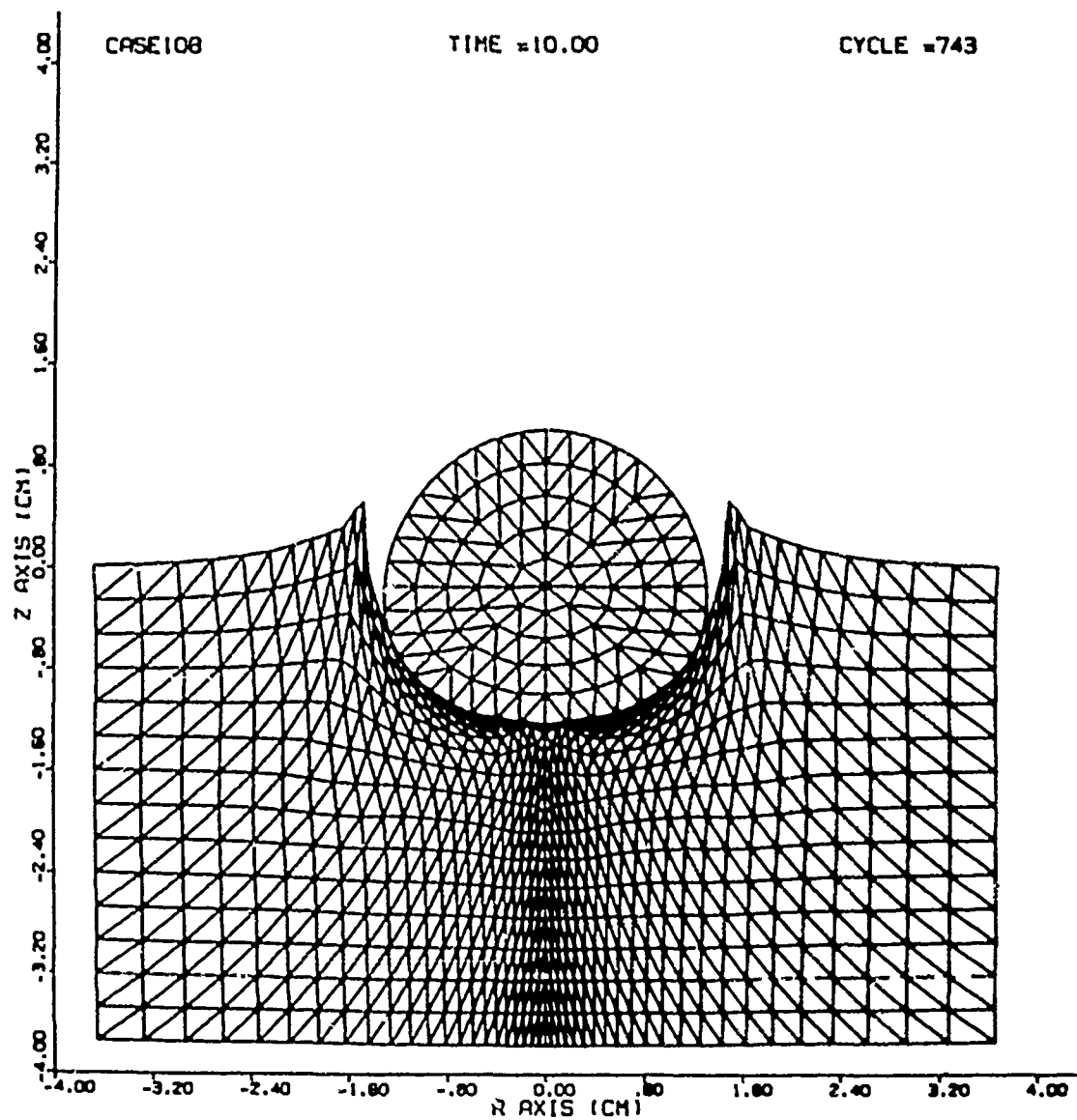


Figure 22. Deformation of Sphere/Plate Impact, $t = 10 \mu s$, IDIA = 2.

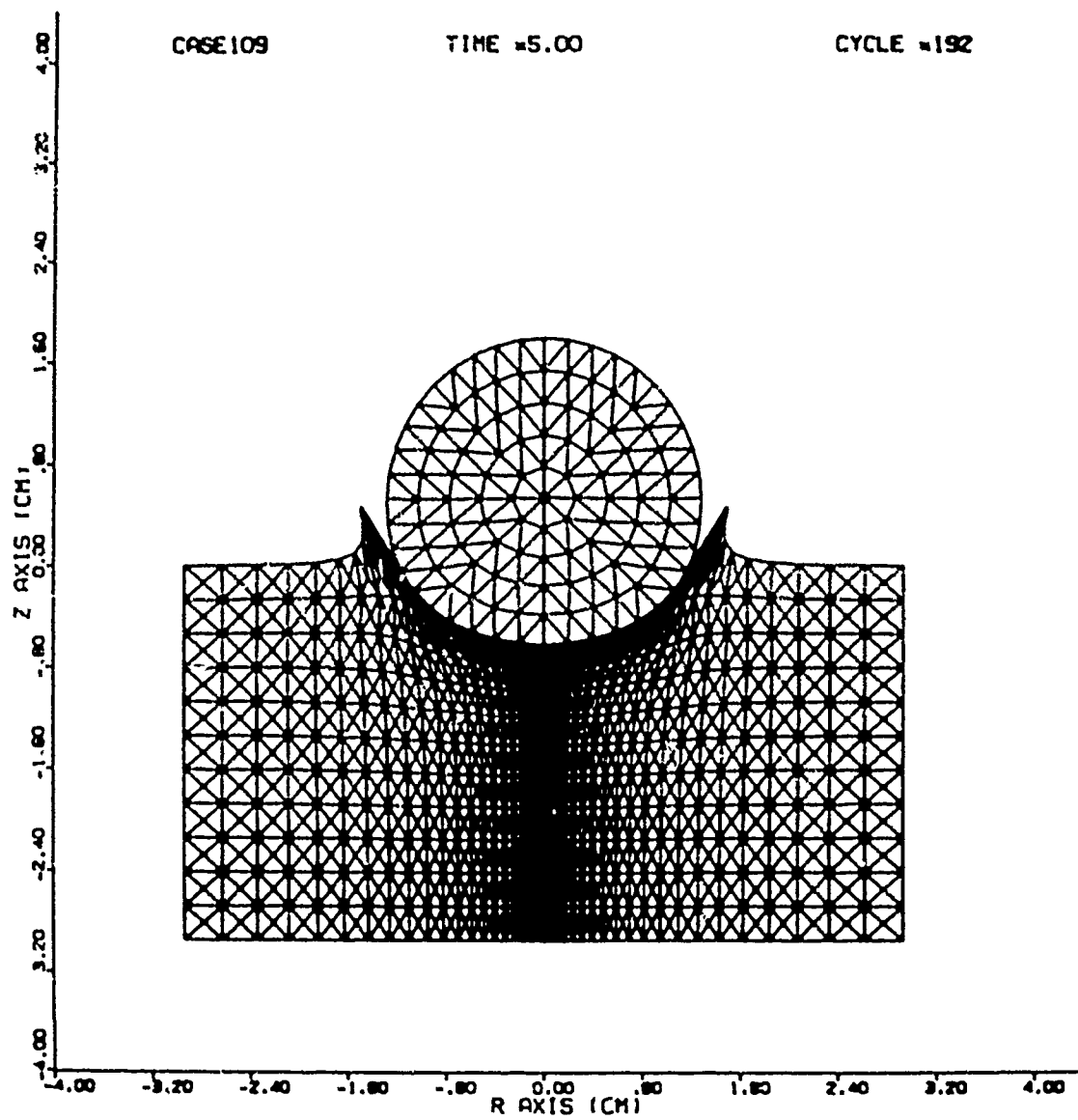


Figure 23. Deformation of Sphere/Plate Impact, $t = 5 \mu s$, Triangles in Quadrilateral Elements.

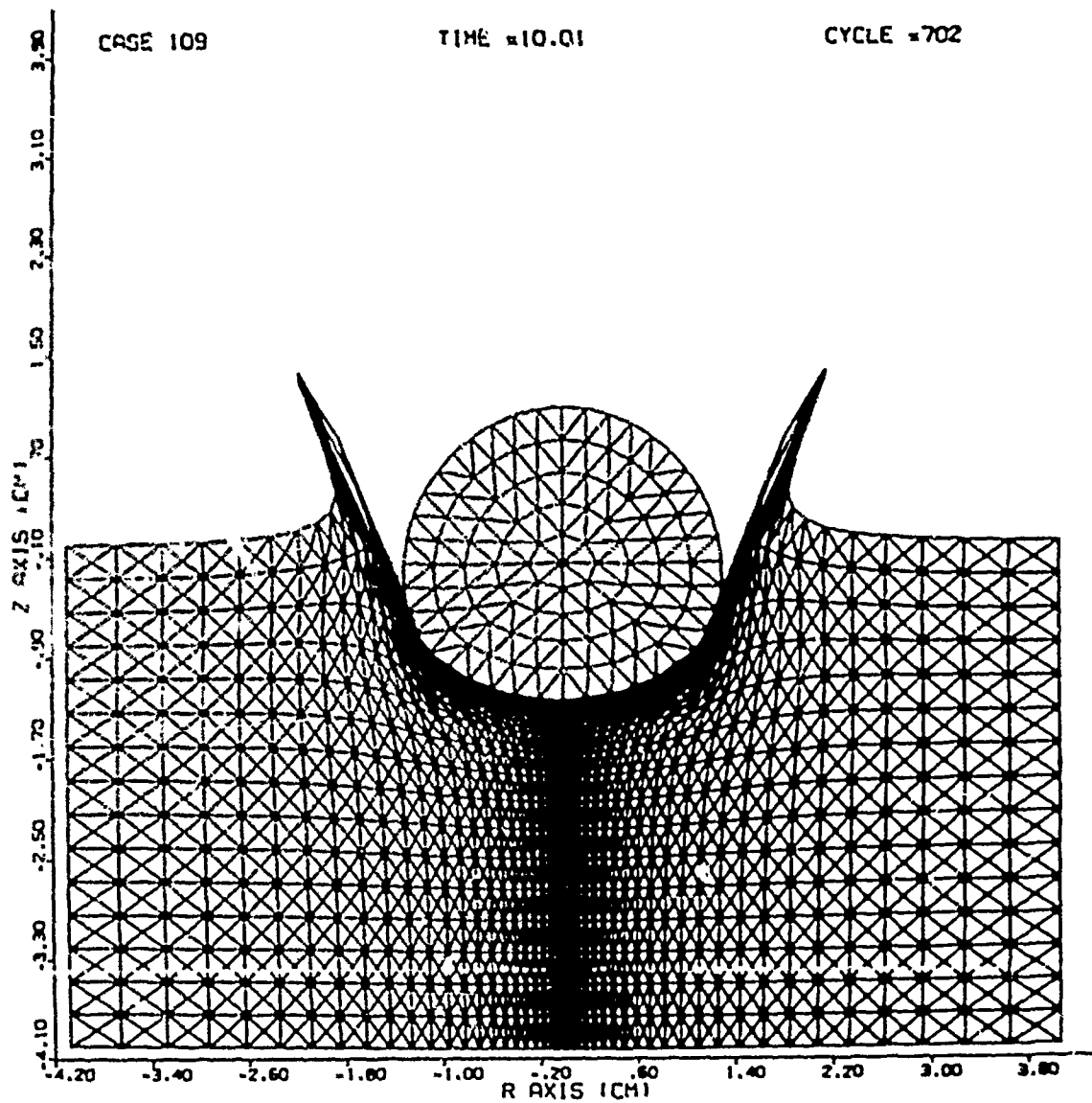


Figure 24. Deformation Sphere/Plate Impact, $t = 10\mu s$, Triangles in Quadrilateral Elements.

the target, and the majority of the pressure fields in the target are located in very small regions.

IV. CONCLUSIONS

The elastic impact analyses demonstrate that close agreement between the analytical (Skalak) and numerical solutions can be obtained provided a fine grid with an aspect ratio of 1:1 is used. Coarse computational grids and high aspect ratios prevent the numerical solution from capturing the high frequency components.

For the effects of artificial viscosity upon wave propagation problems, the linear term provides the additional dissipation required to obtain a smooth, continuous solution, but it also tends to broaden the shock front. The linear component should range between the values of 0.05 to 0.20 for the best results. In most computer programs, the linear and quadratic components are used in conjunction with each other in order to stabilize the effects of excessive numerical ringing. The quadratic component is most effective when the values range between 2.0 and 4.0 for the purpose of obtaining accurate results.

In the cases studied, for the high velocity impact of a sphere into a flat plate, the effects upon the failure of the elements, the deformation behavior characteristics of the elements, and the pressure and strain fields show that quadrilateral elements comprised of four triangles per quadrilateral is the most reasonable representation of observed behavior in elastic-plastic impact problems.

APPENDIX A

SPHERE/PLATE IMPACT
FAILED ELEMENT

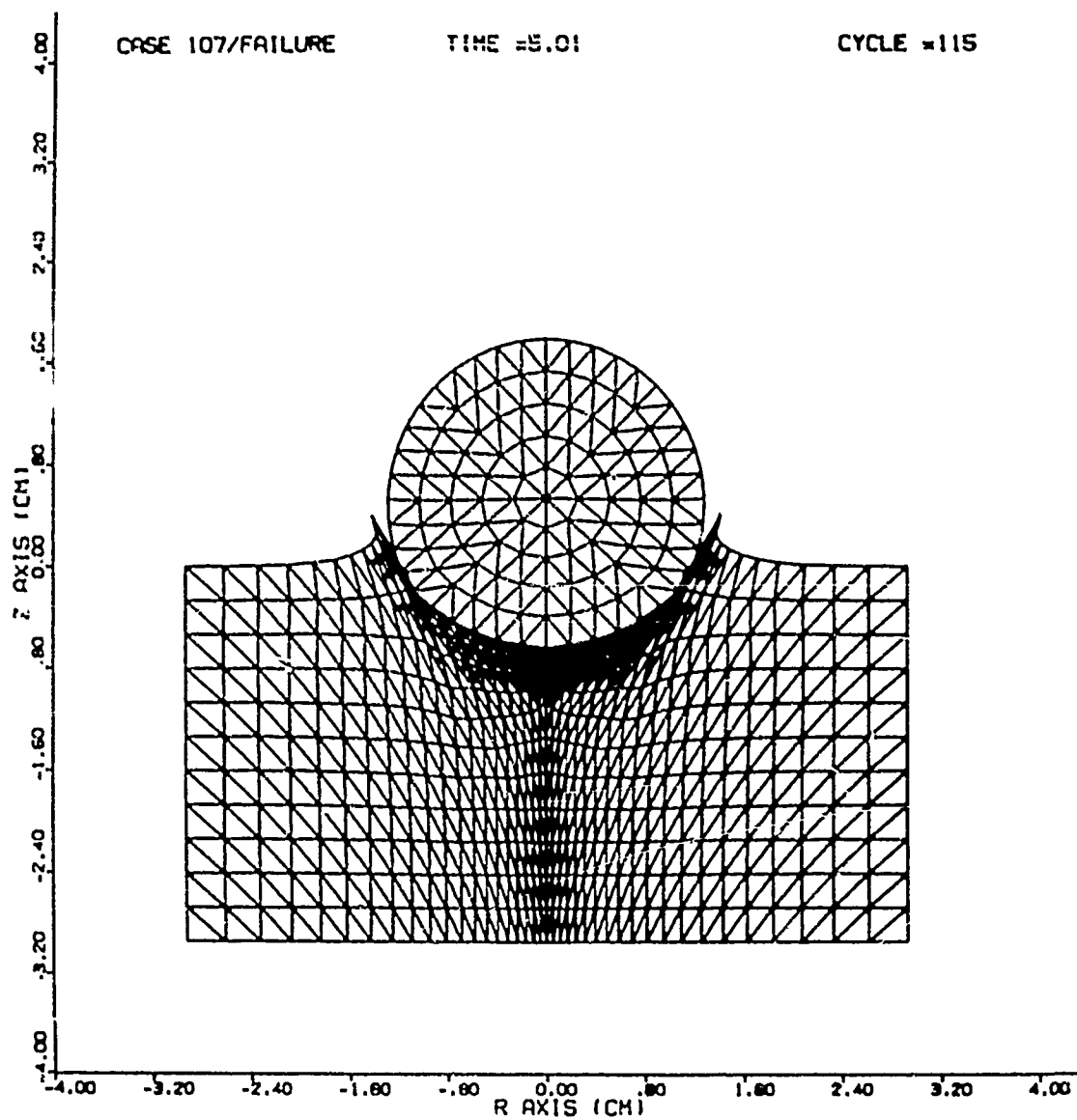


Figure A1. Failed Elements, $t = 5 \mu s$, Case 107, IDIA = 1.

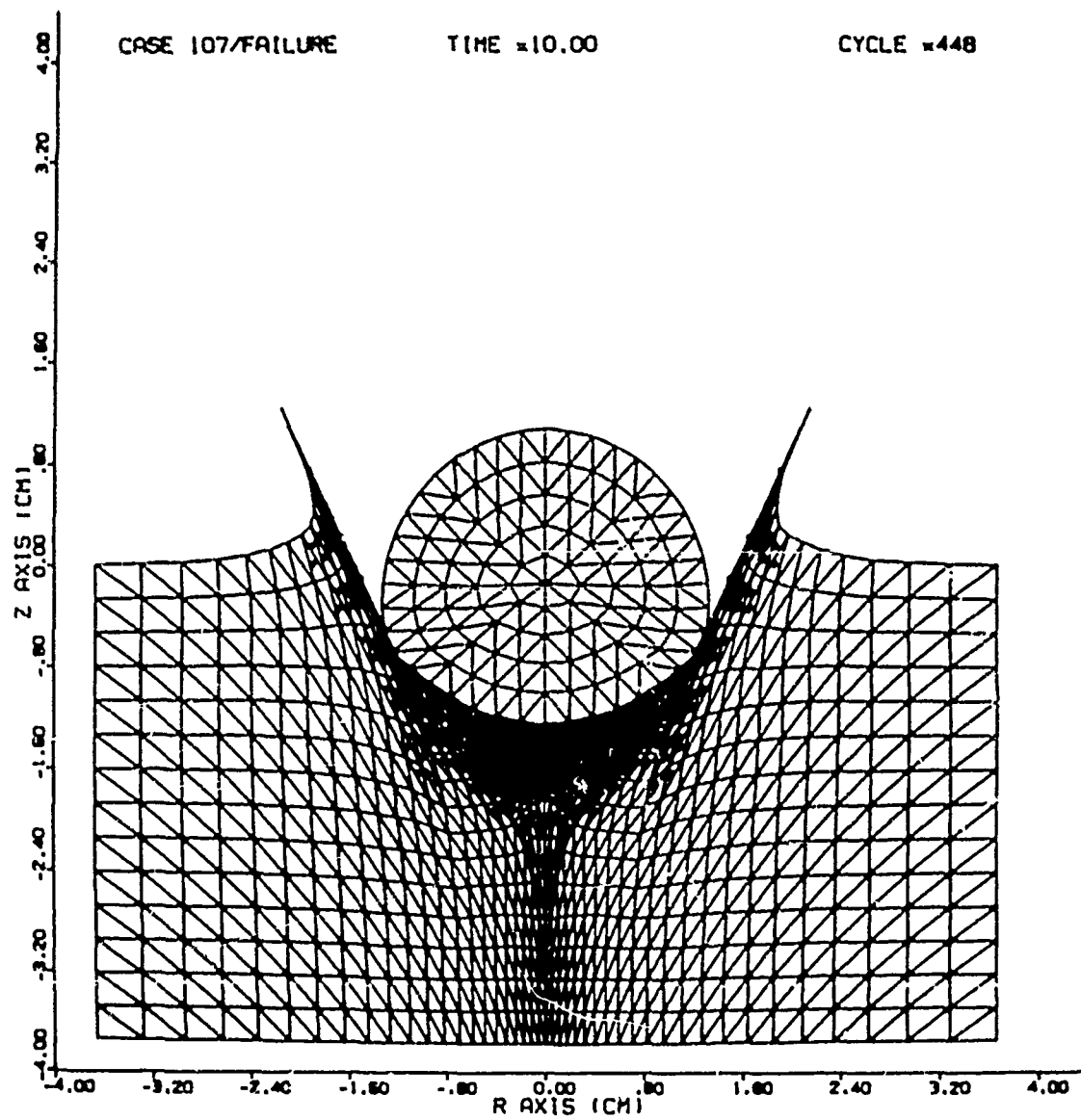


Figure A2. Failed Elements, $t = 10\mu s$, Case 107, IDIA = 1.

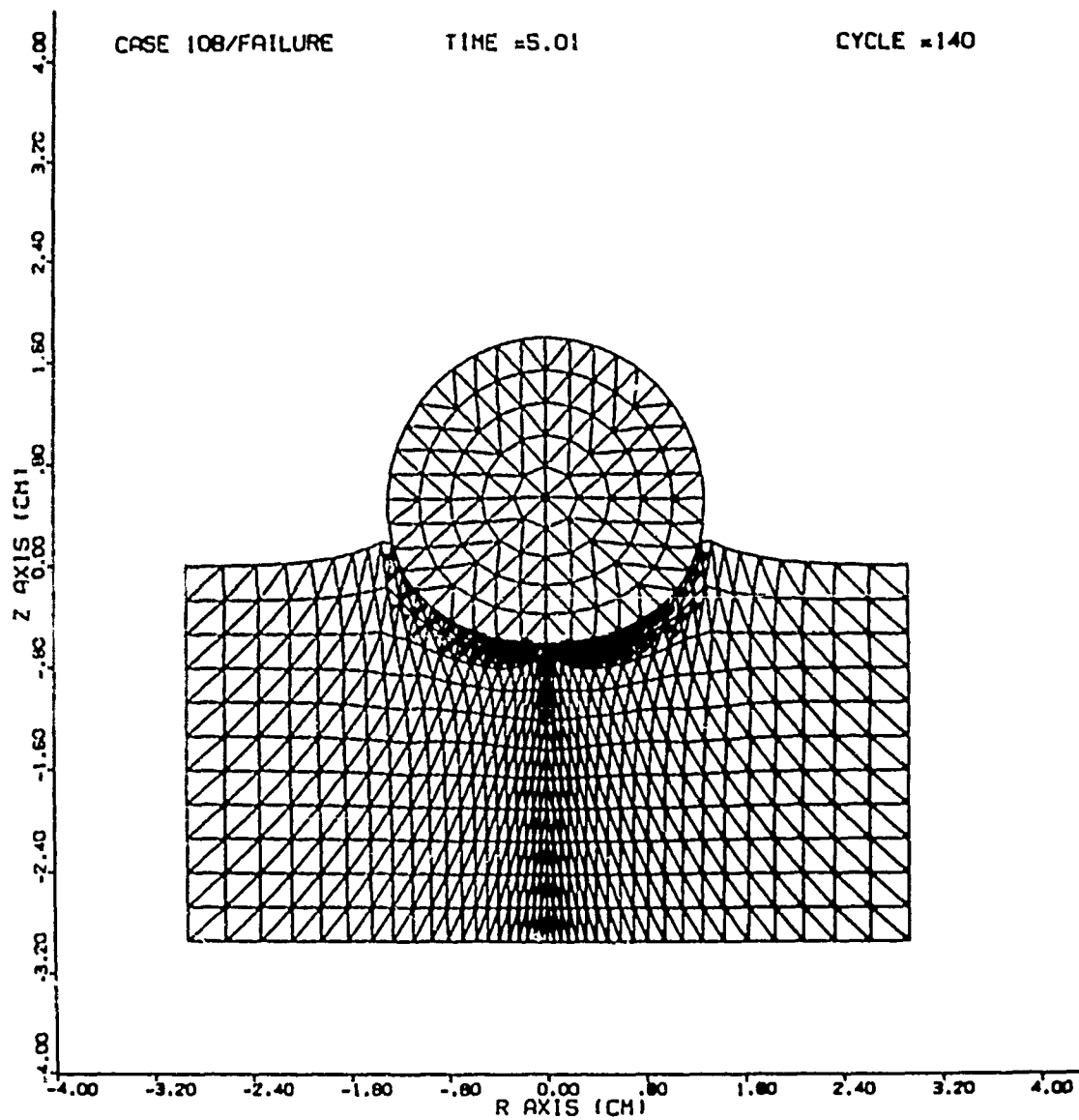


Figure A3. Failed Elements, $t = 5\mu s$, Case 108, IDIA = 2.

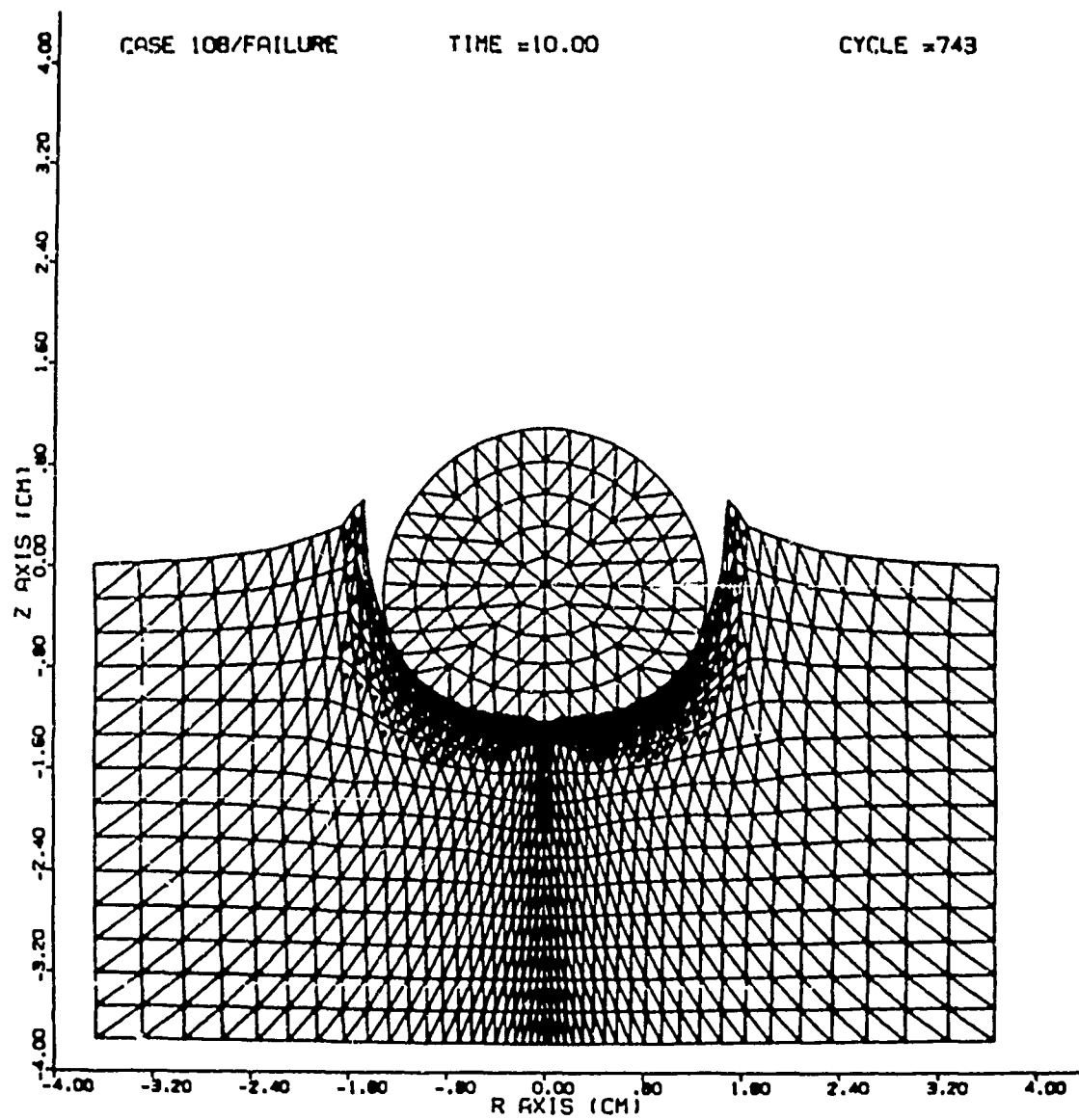


Figure A4. Failed Elements, $t = 10\mu s$, Case 108, IDIA = 2.

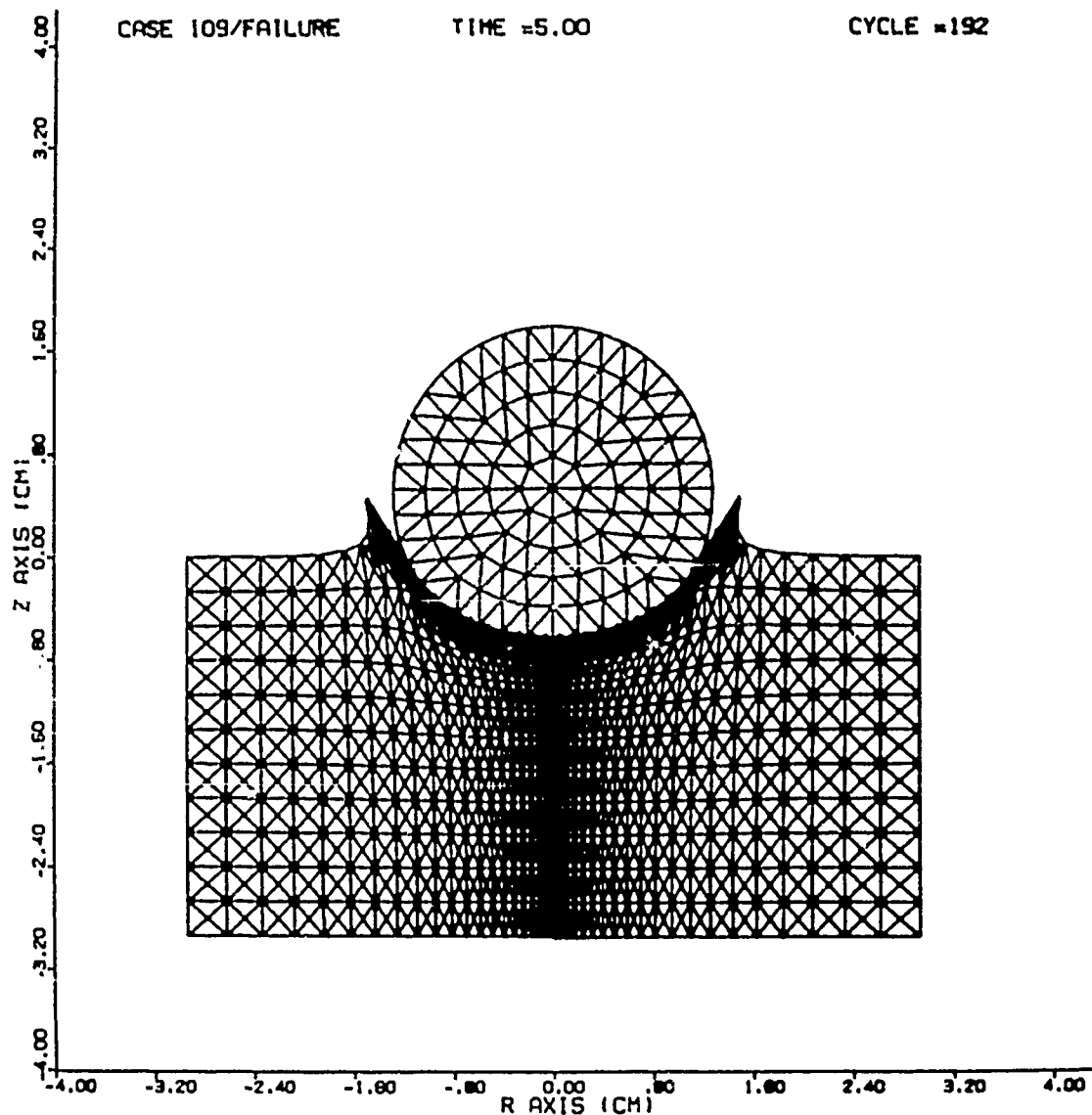


Figure A5. Failed Elements, $t = 5\mu s$, Case 109, Triangles in Quadrilateral Elements.

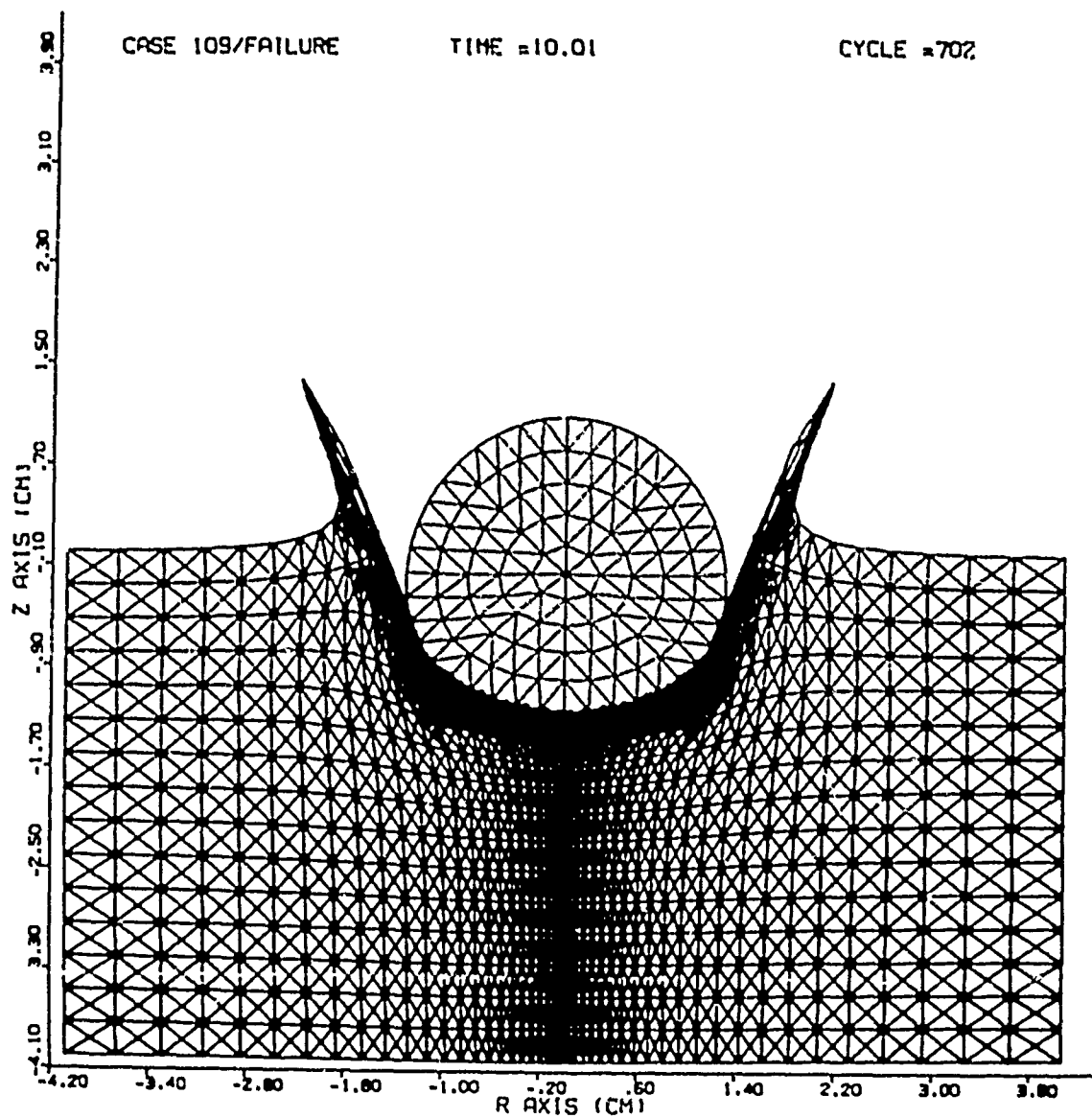


Figure A6. Failed Elements, $T = 10\mu s$, Case 109, Triangles in Quadrilateral Elements.

APPENDIX B

SPHERE/PLATE IMPACT
STRAIN CONTOUR MAPS

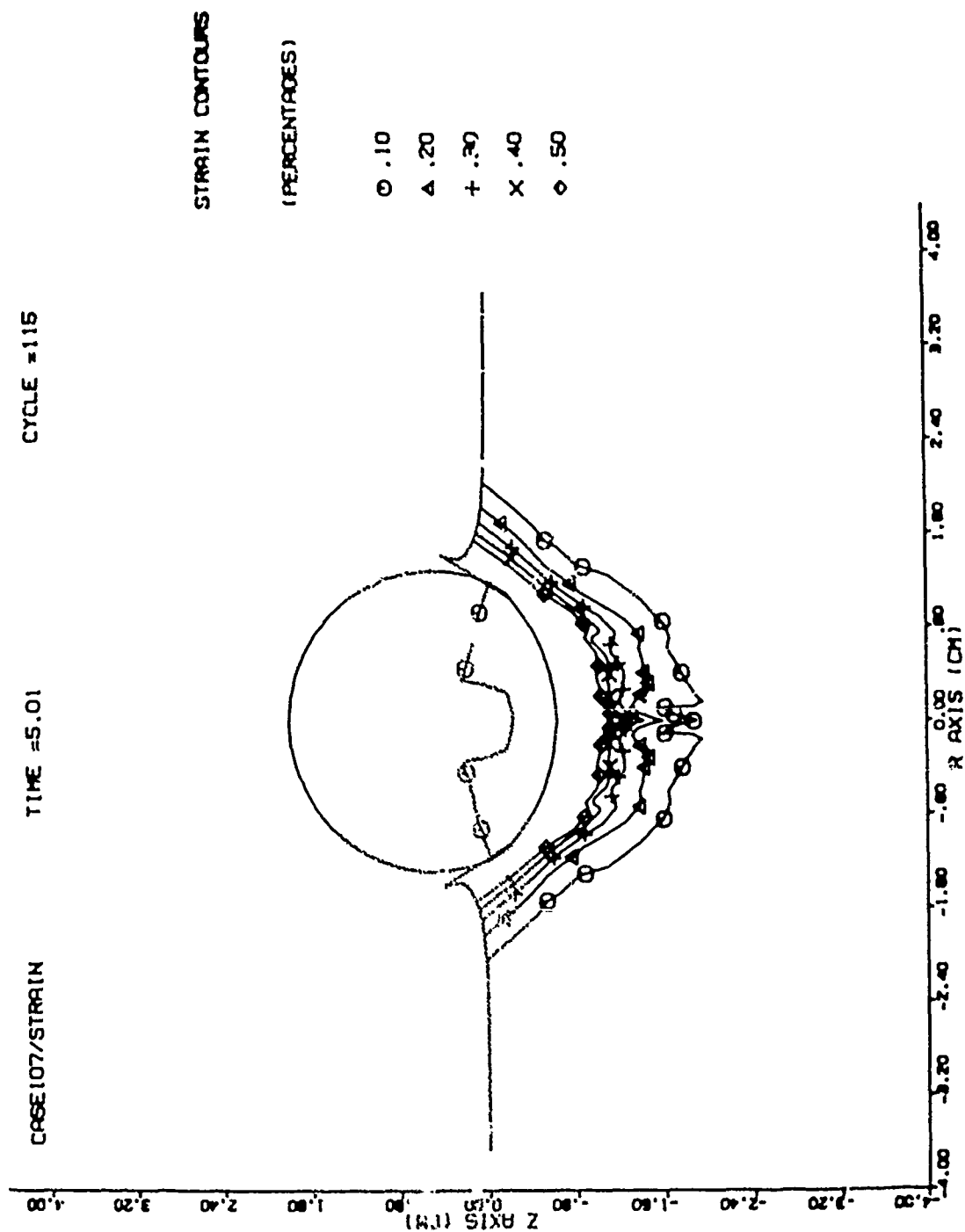


Figure B1. Strain Contour Map, $t = 5\mu s$, Case 107.

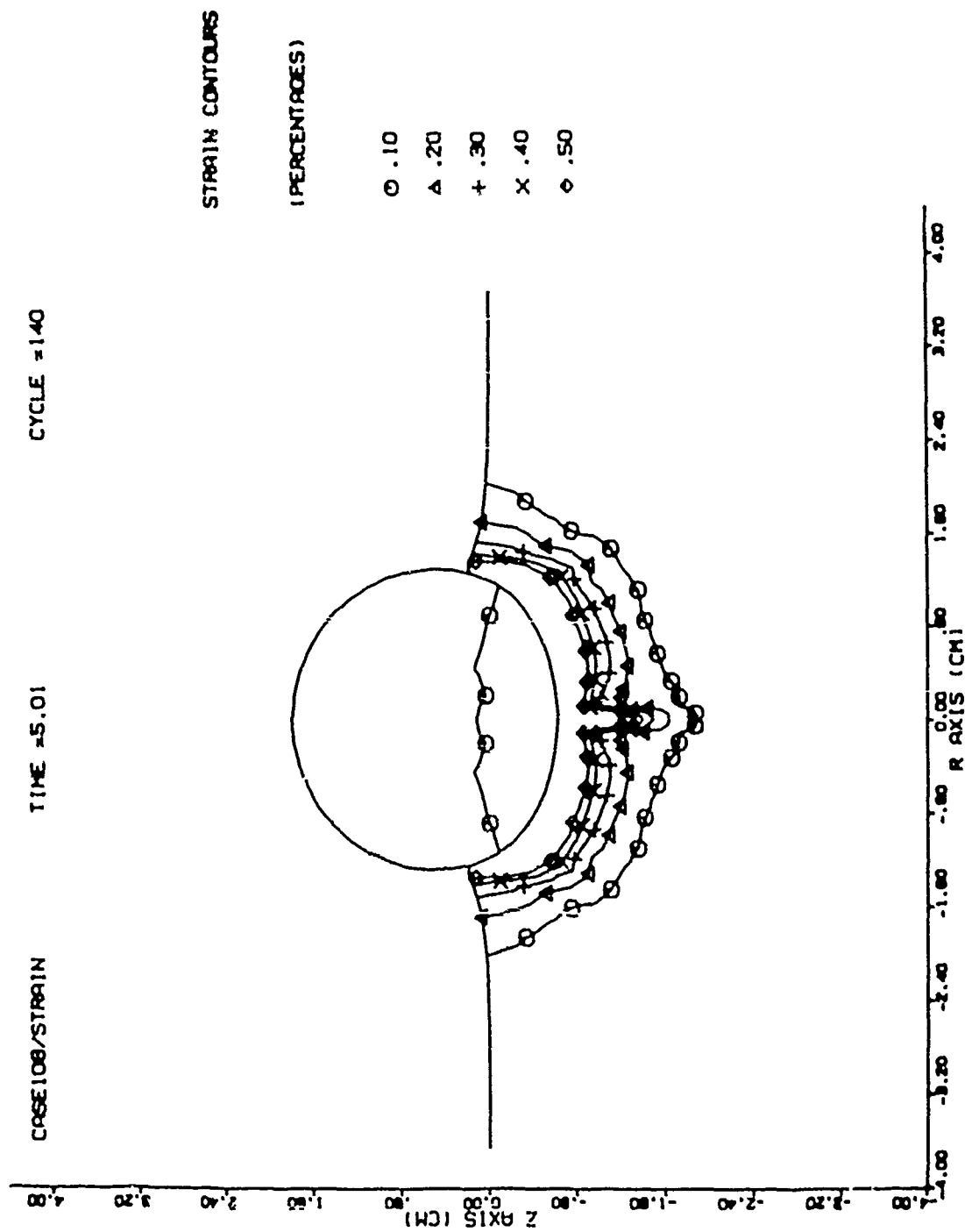


Figure B2. Strain Contour Map, $t = 5\mu s$, Case 103.

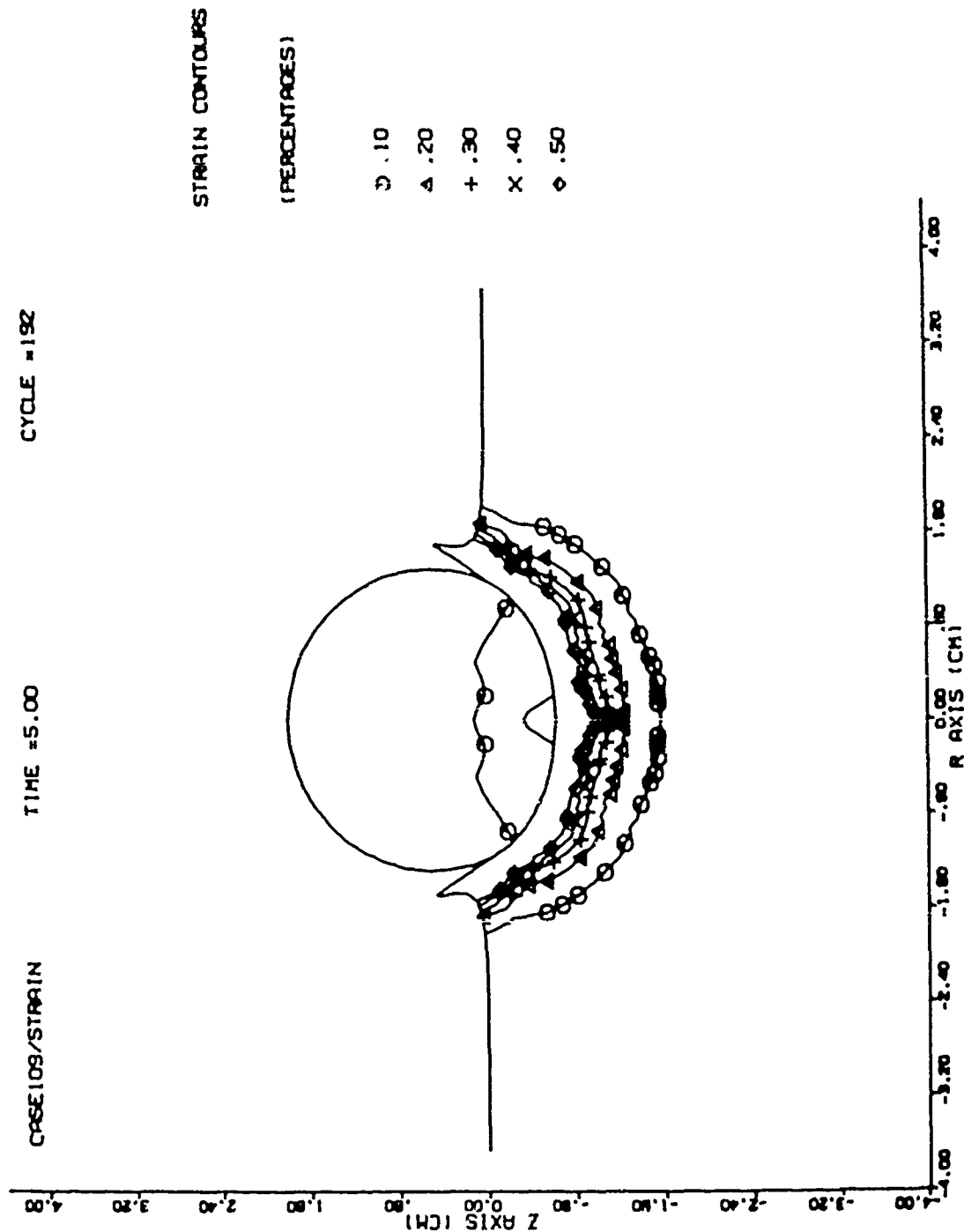


Figure E3. Strain Contour Map, $t = 5\mu s$, Case 109.

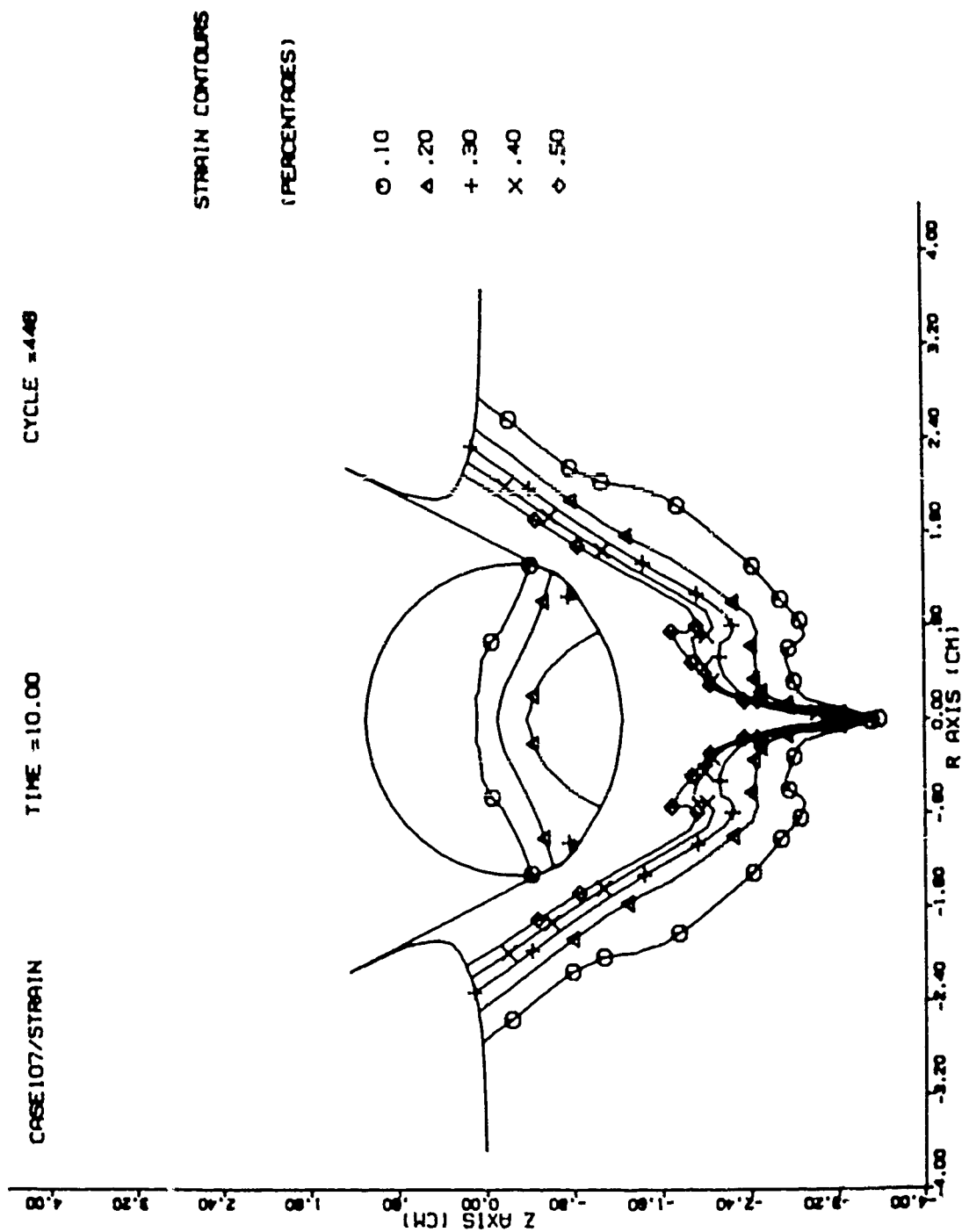


Figure B4. Strain Contour Map, $t = 10\mu s$, Case 107.

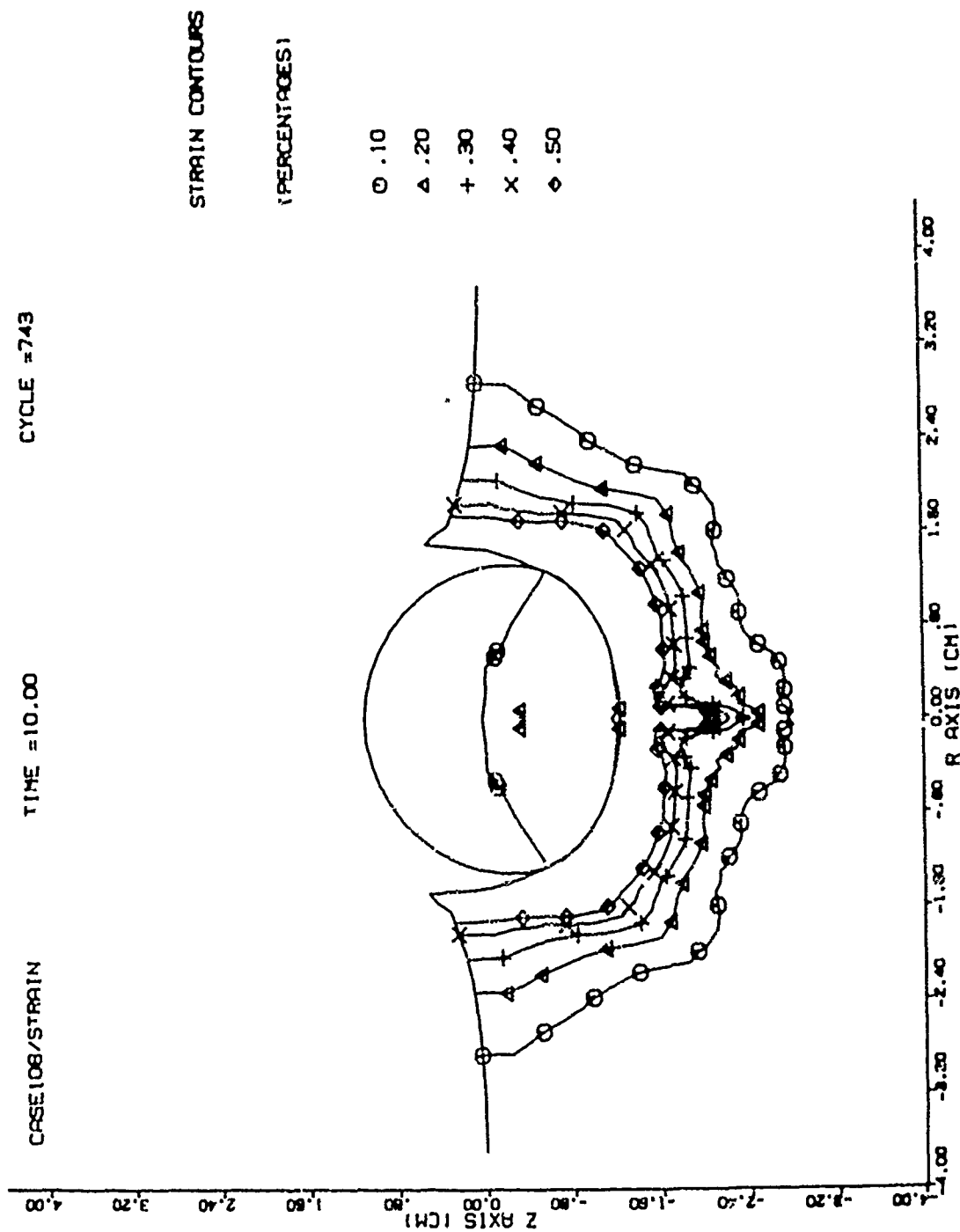


Figure B5. Strain Contour Map, $t = 10\mu s$, Case 108.

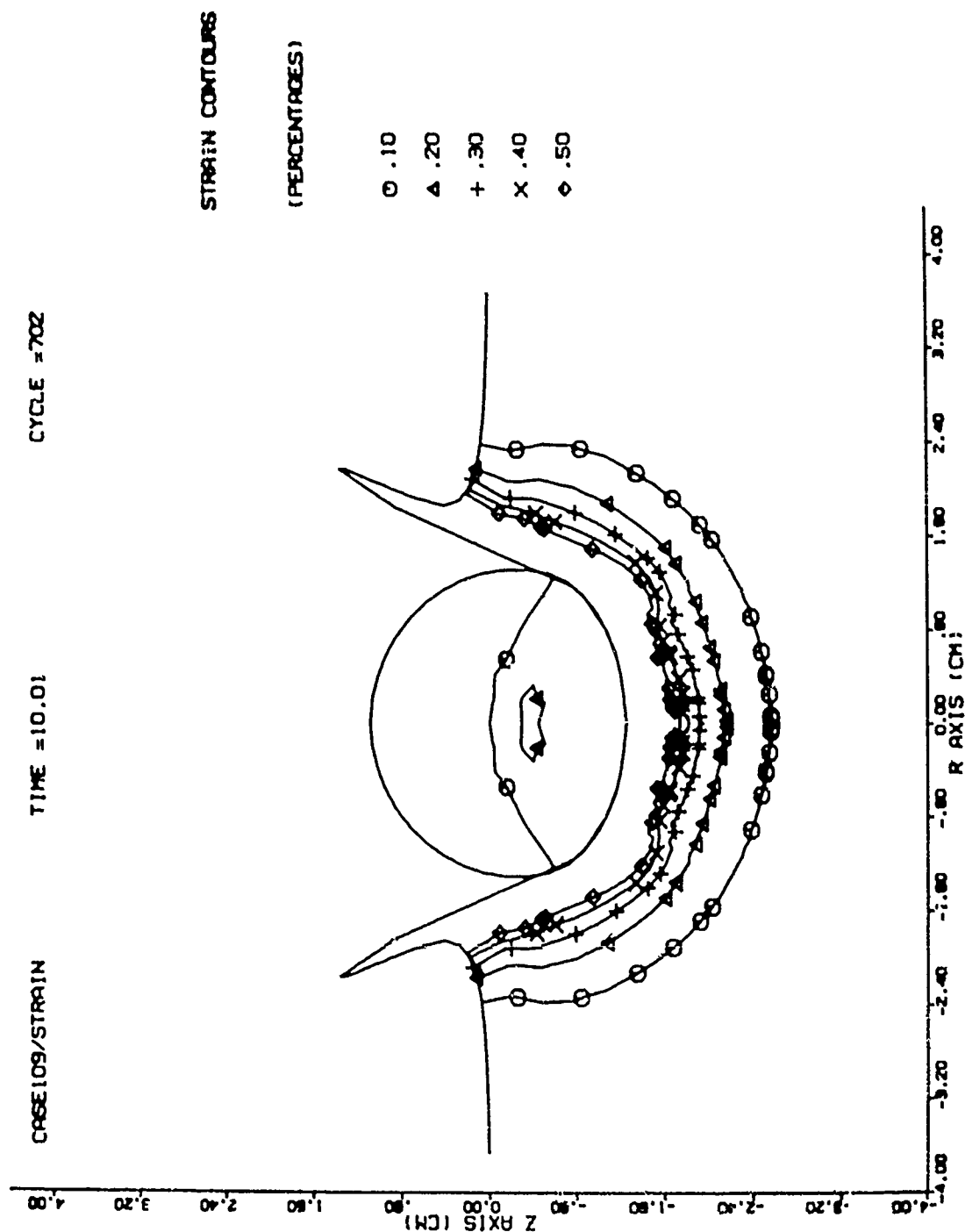


Figure B6. Strain Contour Map, $t = 10\mu s$, Case 109.

APPENDIX C

SPHERE/PLATE IMPACT
PRESSURE/NEGATIVE PRESSURE CONTOUR MAPS

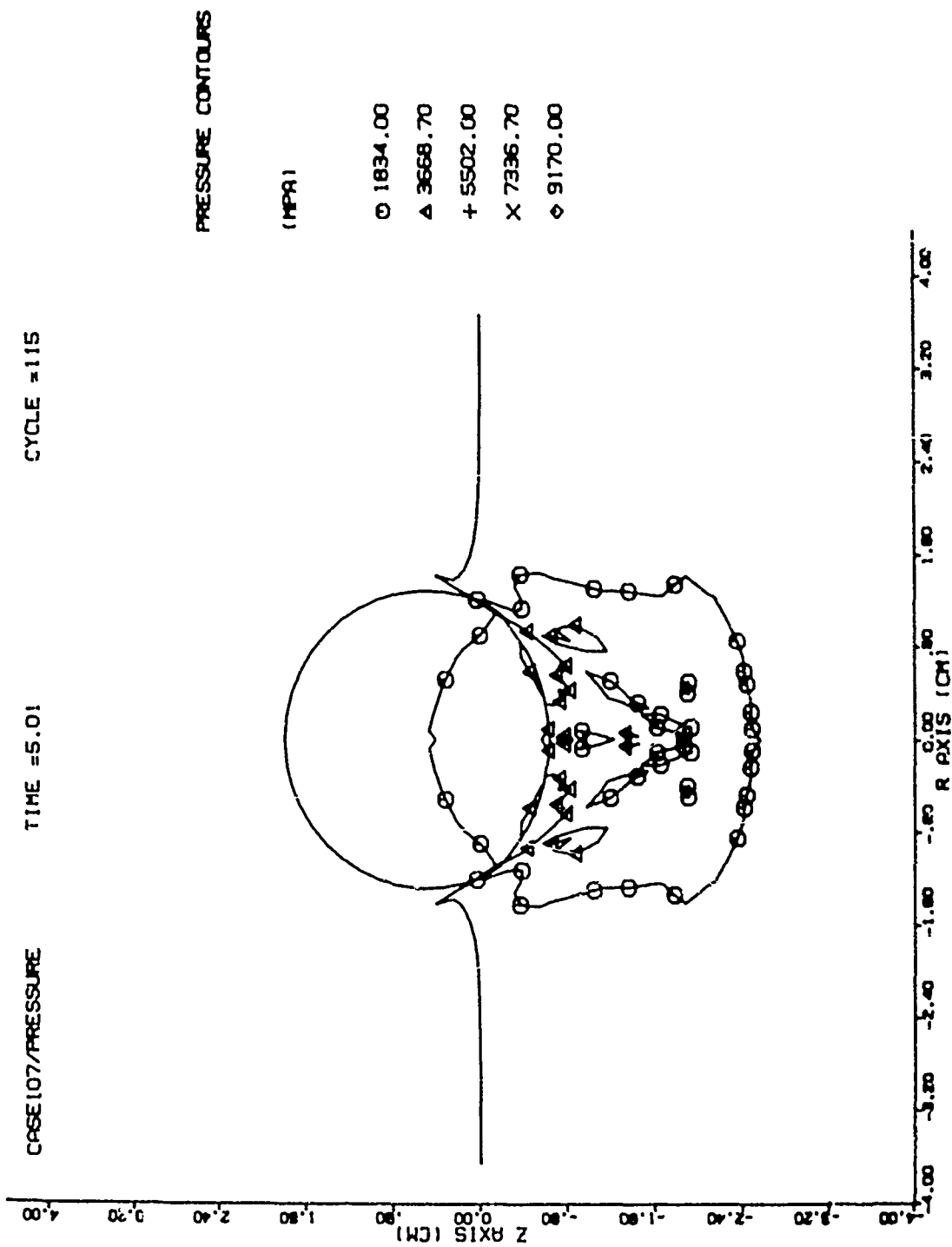


Figure C1. Pressure Contour Map, $t = 5\mu s$, Case 107.

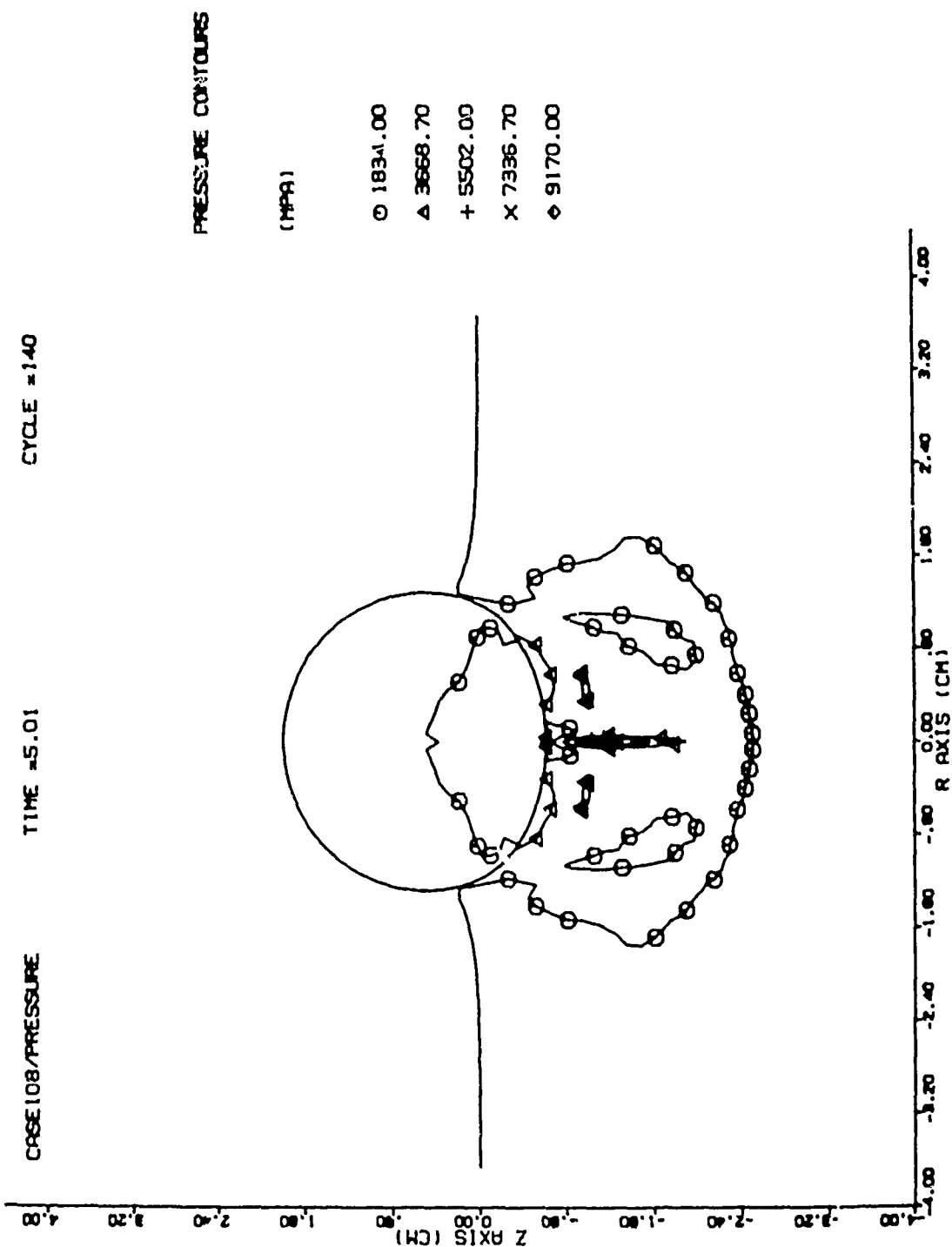


Figure C2. Pressure Contour Map, $t = 5\mu s$, Case 108.

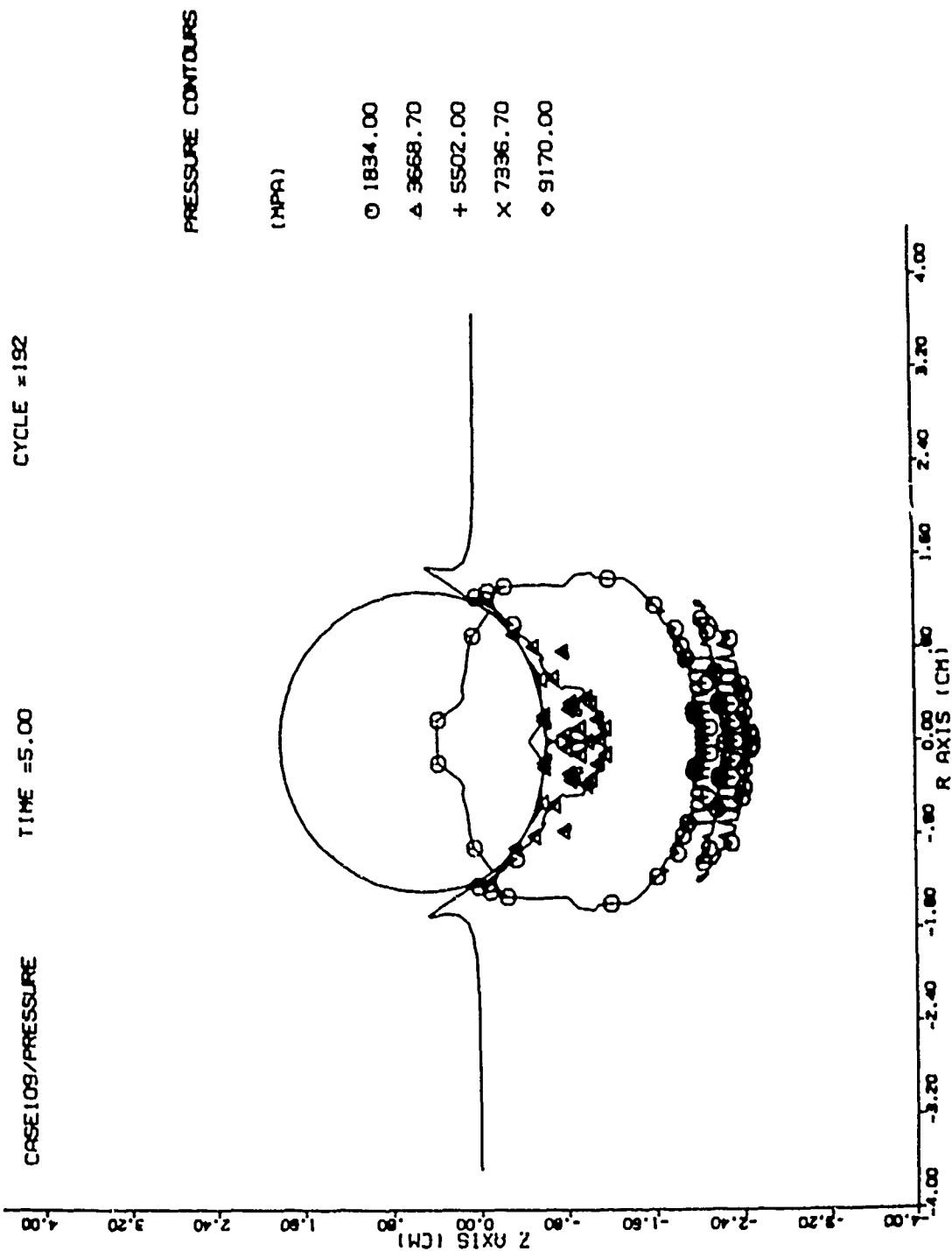


Figure C3. Pressure Contour Map, $t = 5\mu s$, Case 109.

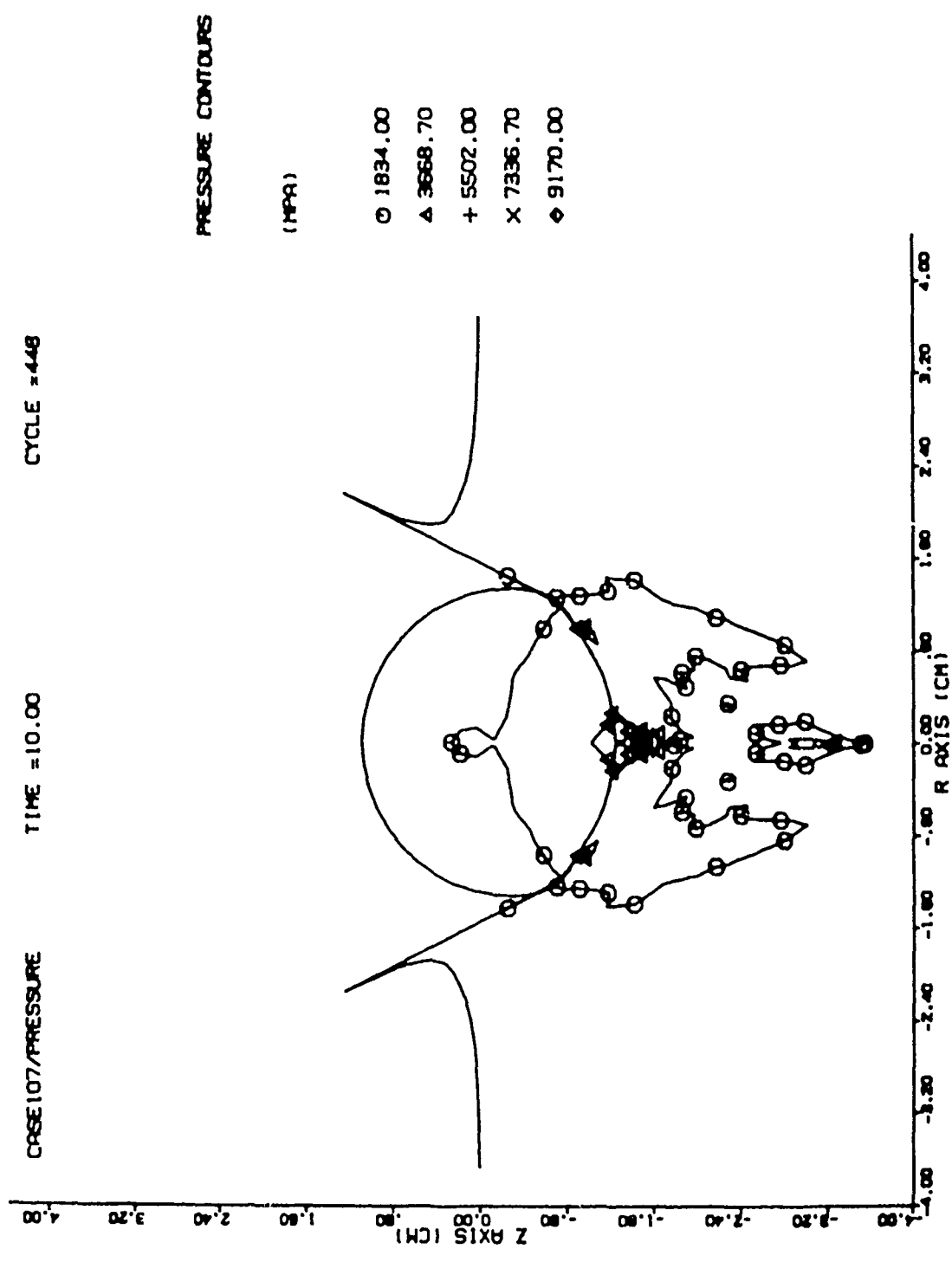


Figure C4. Pressure Contour Map, $t = 10\mu s$, Case 107.

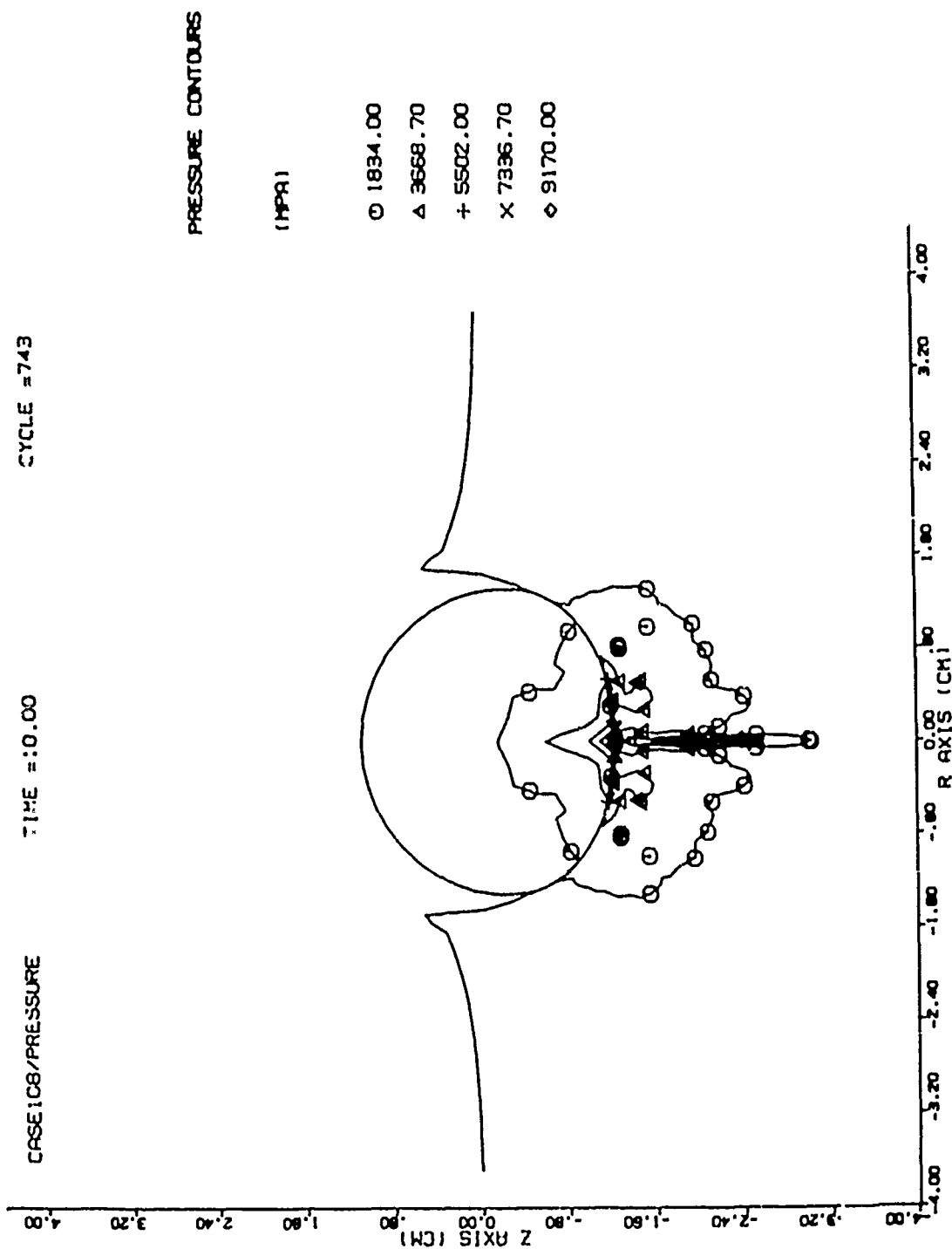
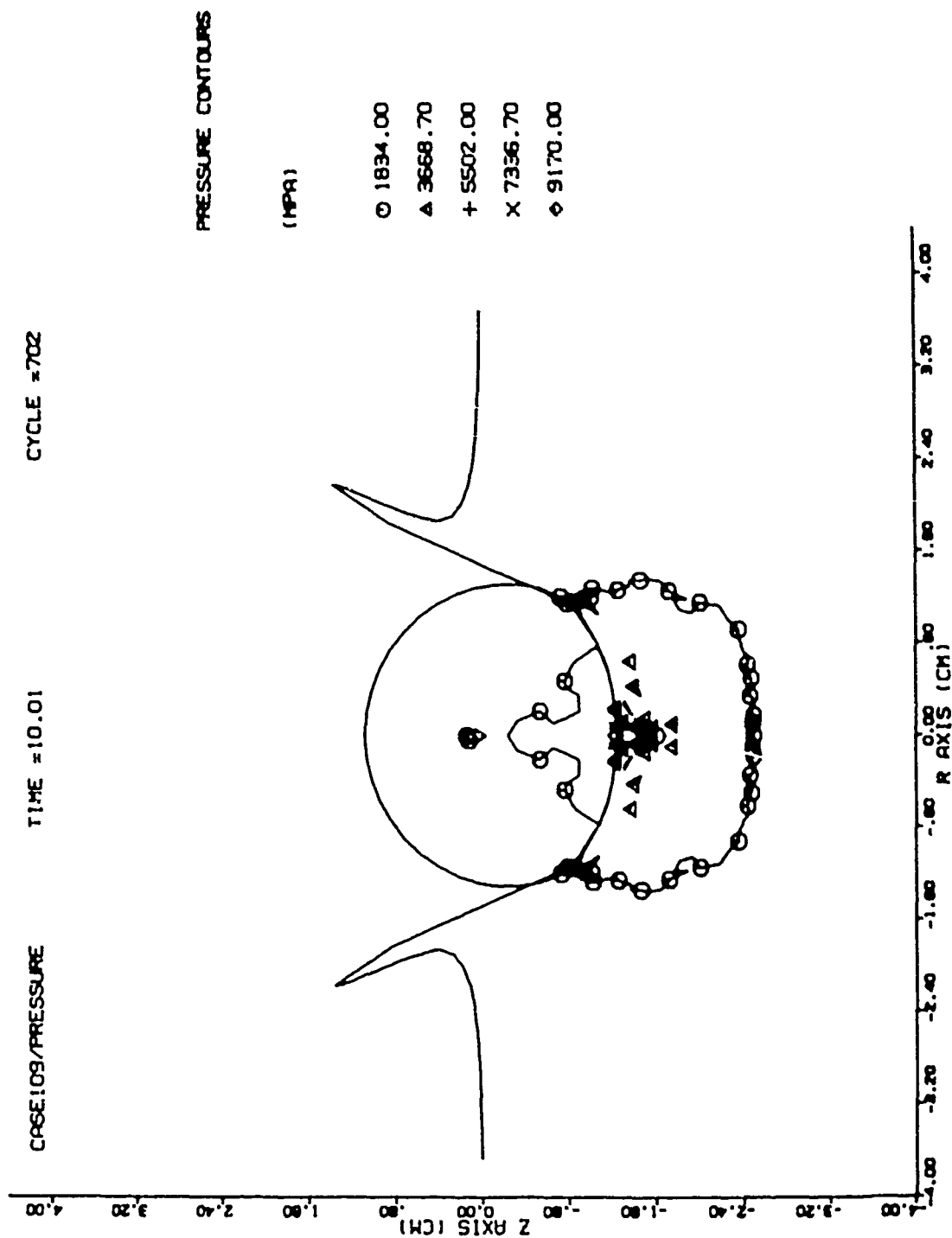


Figure C5. Pressure Contour Map, $t = 10\mu s$, Case 108.



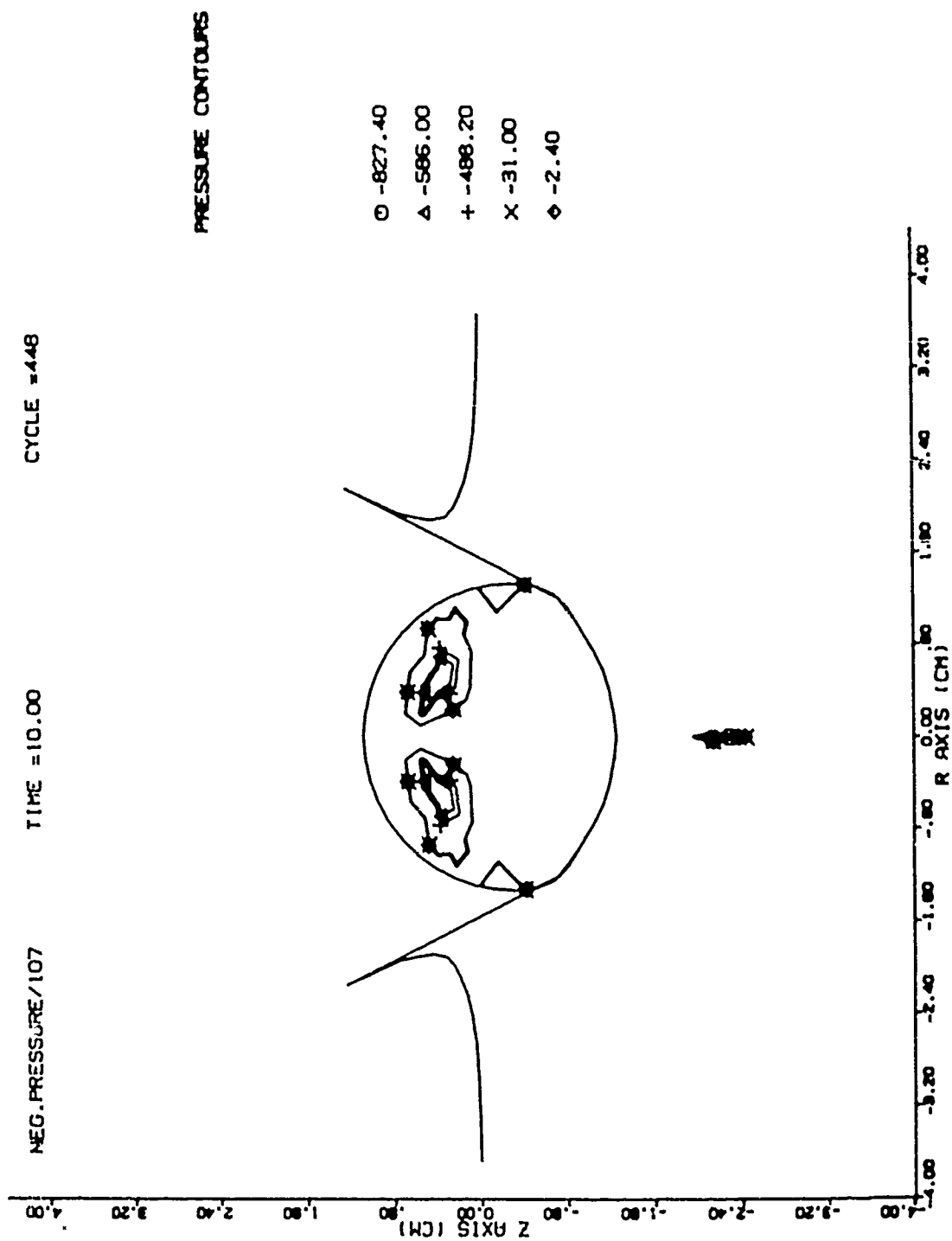


Figure C7. Negative Pressure Contour Map, $t = 10\mu s$, Case 107.

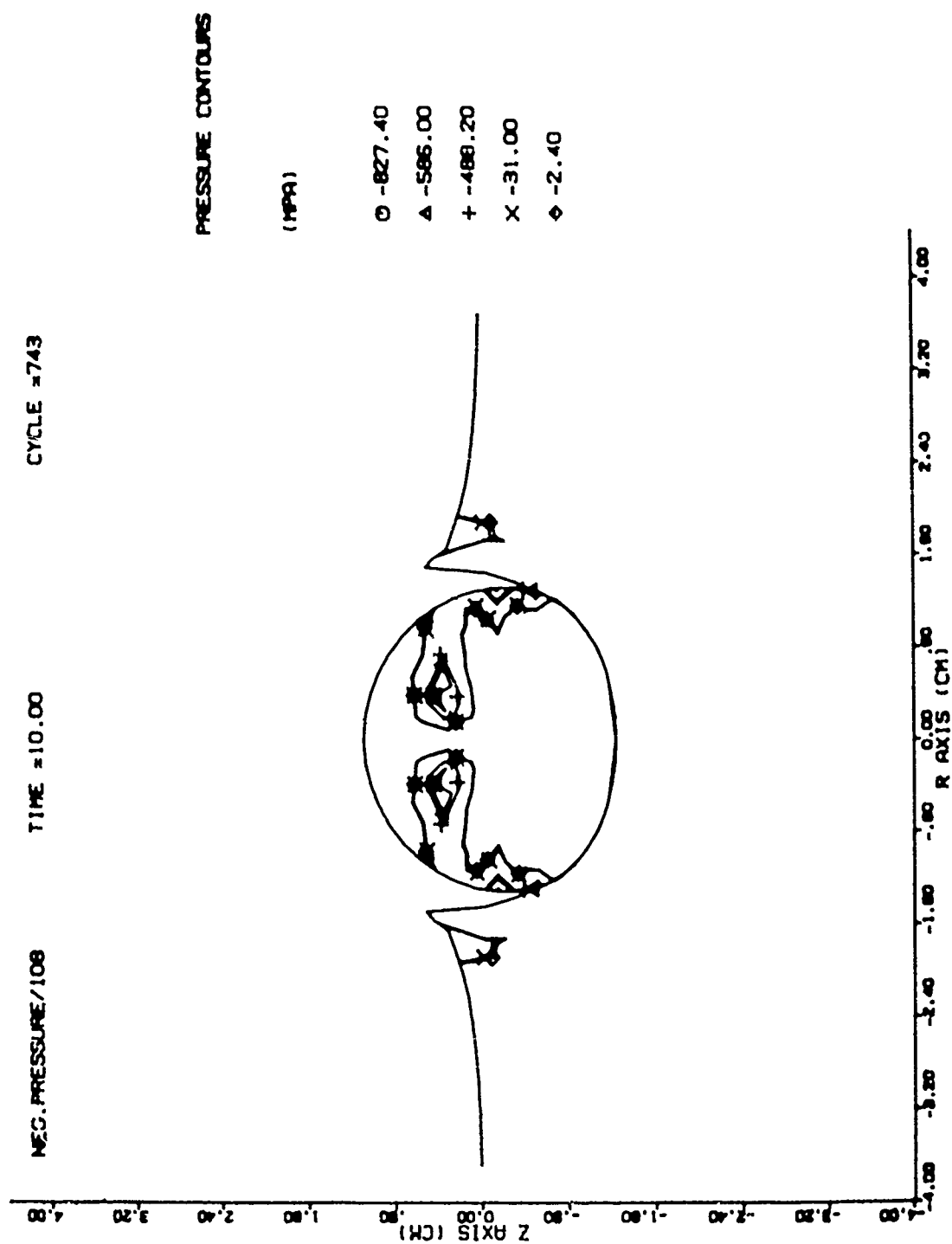


Figure C8. Negative Pressure Contour Map, $t = 10\mu s$, Case 103.

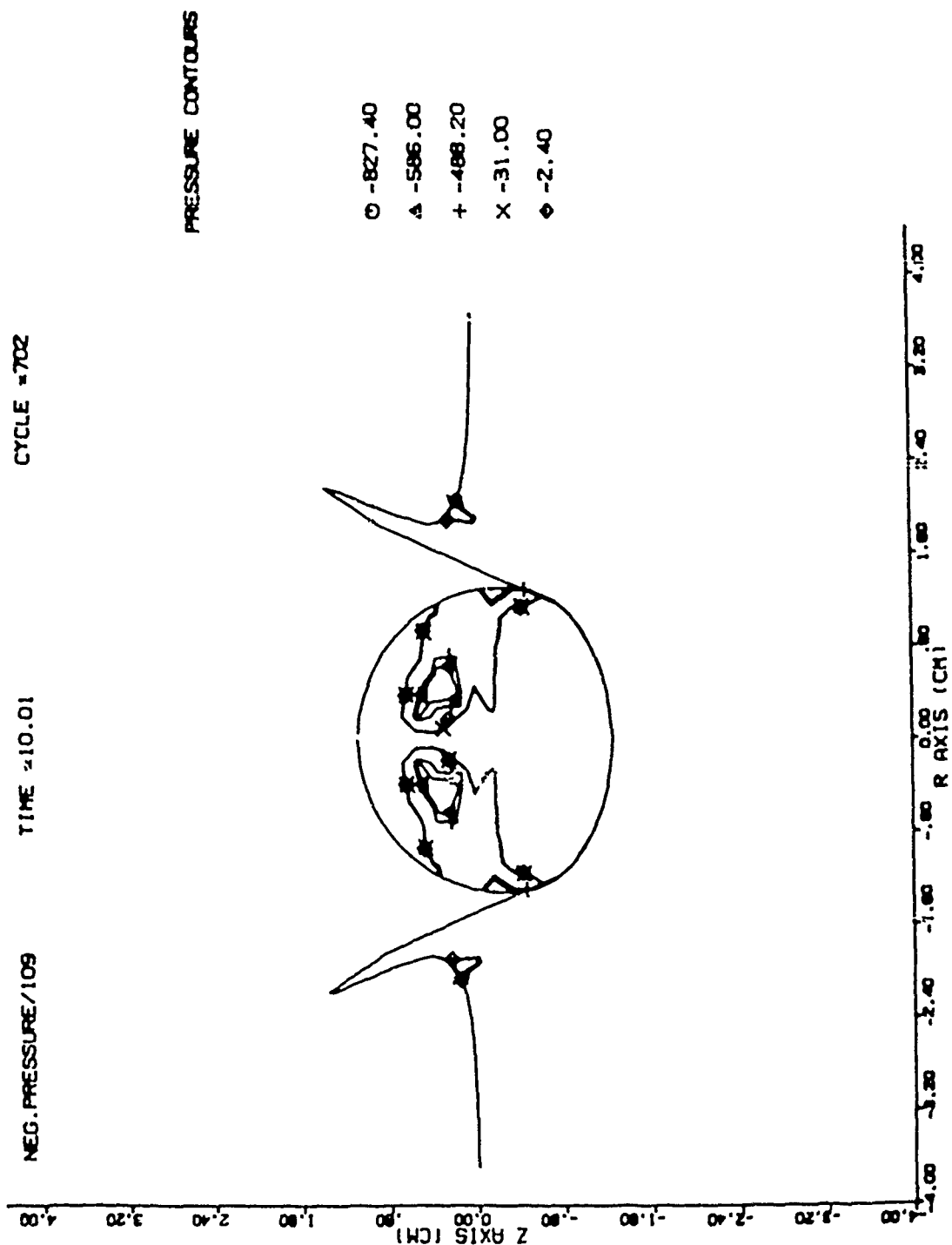


Figure C9. Negative Pressure Contour Map, $t = 10\mu s$, Case 109.

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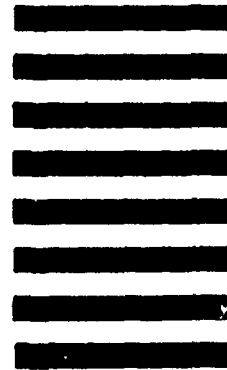


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